# Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "6 Hyperbolic functions\6.2 Hyperbolic cosine"

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## Test results for the 183 problems in "6.2.1 (c+d x)^m (a+b cosh)^n.m"

### Problem 28: Result more than twice size of optimal antiderivative.

$$\int (c+d x) \operatorname{Sech}[a+b x] \, dx$$

Optimal (type 4, 61 leaves, 5 steps):

$$\frac{2 (c+d x) \operatorname{ArcTan}\left[e^{a+b x}\right]}{b}-\frac{i d \operatorname{PolyLog}\left[2,-i e^{a+b x}\right]}{b^2}+\frac{i d \operatorname{PolyLog}\left[2, i e^{a+b x}\right]}{b^2}$$

Result (type 4, 132 leaves):

$$\frac{1}{2 b^2}\left(4 b c \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2}(a+b x)\right]\right]-d(-2 i a+\pi-2 i b x)\left(\operatorname{Log}\left[1-i e^{a+b x}\right]-\operatorname{Log}\left[1+i e^{a+b x}\right]\right)+ d(-2 i a+\pi) \operatorname{Log}\left[\operatorname{Cot}\left[\frac{1}{4}(2 i a+\pi+2 i b x)\right]\right]-2 i d\left(\operatorname{PolyLog}\left[2,-i e^{a+b x}\right]-\operatorname{PolyLog}\left[2, i e^{a+b x}\right]\right)\right)$$

### Problem 32: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c+d x)^2 \operatorname{Sech}[a+b x]^2 \, dx$$

Optimal (type 4, 73 leaves, 5 steps):

$$\frac{(c+d x)^2}{b}-\frac{2 d (c+d x) \operatorname{Log}\left[1+e^{2 (a+b x)}\right]}{b^2}-\frac{d^2 \operatorname{PolyLog}\left[2,-e^{2 (a+b x)}\right]}{b^3}+\frac{(c+d x)^2 \operatorname{Tanh}[a+b x]}{b}$$

Result (type 4, 277 leaves):

$$-\frac{2\,c\,d\,\text{Sech}[a]\,\left(\text{Cosh}[a]\,\text{Log}[\text{Cosh}[a]\,\text{Cosh}[b\,x]+\text{Sinh}[a]\,\text{Sinh}[b\,x]]-b\,x\,\text{Sinh}[a]\right)}{b^2\,\left(\text{Cosh}[a]^2-\text{Sinh}[a]^2\right)}+$$

$$\left(d^2\,\text{Csch}[a]\,\left(-b^2\,e^{-\text{ArcTanh}[\text{Coth}[a]]}\,x^2+\frac{1}{\sqrt{1-\text{Coth}[a]^2}}\,i\,\text{Coth}[a]\right.\right.$$
$$\left(-b\,x\,\left(-\pi+2\,i\,\text{ArcTanh}[\text{Coth}[a]]\right)-\pi\,\text{Log}\left[1+e^{2\,b\,x}\right]-2\,\left(i\,b\,x+i\,\text{ArcTanh}[\text{Coth}[a]]\right)\,\text{Log}\left[1-e^{2\,i\,(i\,b\,x+i\,\text{ArcTanh}[\text{Coth}[a]])}\right]+\right.$$
$$\left.\left.\left.\pi\,\text{Log}[\text{Cosh}[b\,x]]+2\,i\,\text{ArcTanh}[\text{Coth}[a]]\,\text{Log}[i\,\text{Sinh}[b\,x+\text{ArcTanh}[\text{Coth}[a]]]]+i\,\text{PolyLog}\left[2,\,e^{2\,i\,(i\,b\,x+i\,\text{ArcTanh}[\text{Coth}[a]])}\right]\right)\right)\,\text{Sech}[a]\right)\Big/$$
$$\left(b^3\,\sqrt{\text{Csch}[a]^2\,\left(-\text{Cosh}[a]^2+\text{Sinh}[a]^2\right)}\right)+\frac{\text{Sech}[a]\,\text{Sech}[a+b\,x]\,\left(c^2\,\text{Sinh}[b\,x]+2\,c\,d\,x\,\text{Sinh}[b\,x]+d^2\,x^2\,\text{Sinh}[b\,x]\right)}{b}$$

# Problem 38: Result more than twice size of optimal antiderivative.

$$\int (c+d\,x)\,\text{Sech}[a+b\,x]^3\,dx$$

Optimal (type 4, 102 leaves, 6 steps):

$$\frac{(c+d\,x)\,\text{ArcTan}\left[e^{a+b\,x}\right]}{b}-\frac{i\,d\,\text{PolyLog}\left[2,\,-i\,e^{a+b\,x}\right]}{2\,b^2}+\frac{i\,d\,\text{PolyLog}\left[2,\,i\,e^{a+b\,x}\right]}{2\,b^2}+\frac{d\,\text{Sech}[a+b\,x]}{2\,b^2}+\frac{(c+d\,x)\,\text{Sech}[a+b\,x]\,\text{Tanh}[a+b\,x]}{2\,b}$$

Result (type 4, 263 leaves):

$$\frac{c\,\text{ArcTan}\left[\text{Tanh}\left[\frac{1}{2}\,(a+b\,x)\right]\right]}{b}-\frac{1}{2\,b^2}$$
$$d\,\left(\left(\left(-i\,a+\frac{\pi}{2}-i\,b\,x\right)\,\left(\text{Log}\left[1-e^{i\,\left(-i\,a+\frac{\pi}{2}-i\,b\,x\right)}\right]-\text{Log}\left[1+e^{i\,\left(-i\,a+\frac{\pi}{2}-i\,b\,x\right)}\right]\right)-\left(-i\,a+\frac{\pi}{2}\right)\,\text{Log}\left[\text{Tan}\left[\frac{1}{2}\,\left(-i\,a+\frac{\pi}{2}-i\,b\,x\right)\right]\right]+\right.\right.$$
$$\left.\left.i\,\left(\text{PolyLog}\left[2,\,-e^{i\,\left(-i\,a+\frac{\pi}{2}-i\,b\,x\right)}\right]-\text{PolyLog}\left[2,\,e^{i\,\left(-i\,a+\frac{\pi}{2}-i\,b\,x\right)}\right]\right)\right)\right)+$$
$$\frac{d\,\text{Sech}[a]\,\text{Sech}[a+b\,x]\,\left(\text{Cosh}[a]+b\,x\,\text{Sinh}[a]\right)}{2\,b^2}+\frac{d\,x\,\text{Sech}[a]\,\text{Sech}[a+b\,x]^2\,\text{Sinh}[b\,x]}{2\,b}+\frac{c\,\text{Sech}[a+b\,x]\,\text{Tanh}[a+b\,x]}{2\,b}$$

# Problem 39: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{Sech}[a+b\,x]^3}{c+d\,x}\,dx$$

Optimal (type 9, 18 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\text{Sech}[a+b\,x]^3}{c+d\,x},\,x\right]$$

Result (type 1, 1 leaves):

???

## Problem 40: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{Sech}[a+b\,x]^3}{(c+d\,x)^2}\,dx$$

Optimal (type 9, 18 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\text{Sech}[a+b\,x]^3}{(c+d\,x)^2},\,x\right]$$

Result (type 1, 1 leaves):

???

## Problem 48: Result more than twice size of optimal antiderivative.

$$\int (c+d\,x)^{5/2}\,\text{Cosh}[a+b\,x]^2\,dx$$

Optimal (type 4, 239 leaves, 10 steps):

$$\frac{5\,d\,(c+d\,x)^{3/2}}{16\,b^2}+\frac{(c+d\,x)^{7/2}}{7\,d}-\frac{5\,d\,(c+d\,x)^{3/2}\,\text{Cosh}[a+b\,x]^2}{8\,b^2}+\frac{15\,d^{5/2}\,e^{-2\,a+\frac{2\,b\,c}{d}}\,\sqrt{\frac{\pi}{2}}\,\text{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]}{256\,b^{7/2}}-\frac{15\,d^{5/2}\,e^{2\,a-\frac{2\,b\,c}{d}}\,\sqrt{\frac{\pi}{2}}\,\text{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]}{256\,b^{7/2}}+\frac{(c+d\,x)^{5/2}\,\text{Cosh}[a+b\,x]\,\text{Sinh}[a+b\,x]}{2\,b}+\frac{15\,d^2\,\sqrt{c+d\,x}\,\text{Sinh}[2\,a+2\,b\,x]}{64\,b^3}$$

Result (type 4, 3531 leaves):

$$\frac{(c+d\,x)^{7/2}}{7\,d}+\frac{1}{2}\,c^2\,\text{Cosh}[2\,a]\left(-\frac{2\left(\frac{d\,\sqrt{c+d\,x}\,\text{Cosh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]}{4\,b}-\frac{d^{3/2}\,\sqrt{\pi}\left(\text{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+\text{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]\right)}{16\,\sqrt{2}\,b^{3/2}}\right)\text{Sinh}\left[\frac{2\,b\,c}{d}\right]}{d}+\right.$$

$$\left.\frac{2\,\text{Cosh}\left[\frac{2\,b\,c}{d}\right]\left(-\frac{d^{3/2}\sqrt{\pi}\left(-\text{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+\text{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]\right)}{16\sqrt{2}\,b^{3/2}}+\frac{d\sqrt{c+d\,x}\,\text{Sinh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]}{4\,b}\right)}{d}\right)+$$

$$c^2\,\text{Cosh}[a]\,\text{Sinh}[a]\left(\frac{2\,\text{Cosh}\left[\frac{2\,b\,c}{d}\right]\left(\frac{d\sqrt{c+d\,x}\,\text{Cosh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]}{4\,b}-\frac{d^{3/2}\sqrt{\pi}\left(\text{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+\text{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]\right)}{16\sqrt{2}\,b^{3/2}}\right)}{d}-\right.$$

$$\left.\frac{2\,\text{Sinh}\left[\frac{2\,b\,c}{d}\right]\left(-\frac{d^{3/2}\sqrt{\pi}\left(-\text{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+\text{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]\right)}{16\sqrt{2}\,b^{3/2}}+\frac{d\sqrt{c+d\,x}\,\text{Sinh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]}{4\,b}\right)}{d}\right)+$$

$$c\,d\,\text{Cosh}[2\,a]\left(\frac{2\,c\left(\frac{d\sqrt{c+d\,x}\,\text{Cosh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]}{4\,b}-\frac{d^{3/2}\sqrt{\pi}\left(\text{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+\text{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]\right)}{16\sqrt{2}\,b^{3/2}}\right)\text{Sinh}\left[\frac{2\,b\,c}{d}\right]}{d^2}-\right.$$

$$\frac{2\,c\,\text{Cosh}\left[\frac{2\,b\,c}{d}\right]\left(-\frac{d^{3/2}\sqrt{\pi}\left(-\text{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+\text{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]\right)}{16\sqrt{2}\,b^{3/2}}+\frac{d\sqrt{c+d\,x}\,\text{Sinh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]}{4\,b}\right)}{d^2}+\frac{1}{32\sqrt{2}\,b^{5/2}\,d}\text{Sinh}\left[\frac{2\,b\,c}{d}\right]\left(3\,d^{3/2}\sqrt{\pi}\,\text{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]-\right.$$

$$\left.\left.3\,d^{3/2}\sqrt{\pi}\,\text{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+4\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}\left(-4\,b\,(c+d\,x)\,\text{Cosh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]+3\,d\,\text{Sinh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]\right)\right)\right)+$$

$$\frac{1}{32\sqrt{2}\,b^{5/2}\,d}\operatorname{Cosh}\left[\frac{2\,b\,c}{d}\right]\left(3\,d^{3/2}\sqrt{\pi}\,\operatorname{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+3\,d^{3/2}\sqrt{\pi}\,\operatorname{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+\right.$$

$$\left.4\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}\left(-3\,d\operatorname{Cosh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]+4\,b\,(c+d\,x)\operatorname{Sinh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]\right)\right)\Bigg)+$$

$$2\,c\,d\operatorname{Cosh}[a]\operatorname{Sinh}[a]\left(-\frac{2\,c\operatorname{Cosh}\left[\frac{2\,b\,c}{d}\right]\left(\frac{d\sqrt{c+d\,x}\operatorname{Cosh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]}{4\,b}-\frac{d^{3/2}\sqrt{\pi}\left(\operatorname{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+\operatorname{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]\right)}{16\sqrt{2}\,b^{3/2}}\right)}{d^2}+\right.$$

$$\frac{2\,c\operatorname{Sinh}\left[\frac{2\,b\,c}{d}\right]\left(-\frac{d^{3/2}\sqrt{\pi}\left(-\operatorname{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+\operatorname{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]\right)}{16\sqrt{2}\,b^{3/2}}+\frac{d\sqrt{c+d\,x}\operatorname{Sinh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]}{4\,b}\right)}{d^2}+\frac{1}{32\sqrt{2}\,b^{5/2}\,d}$$

$$\operatorname{Cosh}\left[\frac{2\,b\,c}{d}\right]\left(-3\,d^{3/2}\sqrt{\pi}\,\operatorname{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+3\,d^{3/2}\sqrt{\pi}\,\operatorname{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+\right.$$

$$\left.4\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}\left(4\,b\,(c+d\,x)\operatorname{Cosh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]-3\,d\operatorname{Sinh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]\right)\right)-$$

$$\frac{1}{32\sqrt{2}\,b^{5/2}\,d}\operatorname{Sinh}\left[\frac{2\,b\,c}{d}\right]\left(3\,d^{3/2}\sqrt{\pi}\,\operatorname{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+3\,d^{3/2}\sqrt{\pi}\,\operatorname{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+\right.$$

$$\left.4\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}\left(-3\,d\operatorname{Cosh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]+4\,b\,(c+d\,x)\operatorname{Sinh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]\right)\right)\Bigg)+$$

$$\frac{1}{2} d^2 \text{Cosh}[2 a] \left( -\frac{2 c^2 \left( \frac{d \sqrt{c+d x} \text{ Cosh}\left[\frac{2 b (c+d x)}{d}\right]}{4 b} - \frac{d^{3/2} \sqrt{\pi} \left( \text{Erf}\left[\frac{\sqrt{2} \sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right] + \text{Erfi}\left[\frac{\sqrt{2} \sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right] \right)}{16 \sqrt{2} b^{3/2}} \right) \text{Sinh}\left[\frac{2 b c}{d}\right]}{d^3} + \right.$$

$$\frac{2 c^2 \text{Cosh}\left[\frac{2 b c}{d}\right] \left( -\frac{d^{3/2} \sqrt{\pi} \left( -\text{Erf}\left[\frac{\sqrt{2} \sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right] + \text{Erfi}\left[\frac{\sqrt{2} \sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right] \right)}{16 \sqrt{2} b^{3/2}} + \frac{d \sqrt{c+d x} \text{ Sinh}\left[\frac{2 b (c+d x)}{d}\right]}{4 b} \right)}{d^3} + \frac{1}{16 \sqrt{2} b^{5/2} d^2}$$

$$c \text{ Sinh}\left[\frac{2 b c}{d}\right] \left( -3 d^{3/2} \sqrt{\pi} \text{ Erf}\left[\frac{\sqrt{2} \sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right] + 3 d^{3/2} \sqrt{\pi} \text{ Erfi}\left[\frac{\sqrt{2} \sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right] + \right.$$

$$\left. 4 \sqrt{2} \sqrt{b} \sqrt{c+d x} \left( 4 b (c+d x) \text{ Cosh}\left[\frac{2 b (c+d x)}{d}\right] - 3 d \text{ Sinh}\left[\frac{2 b (c+d x)}{d}\right] \right) \right) -$$

$$\frac{1}{16 \sqrt{2} b^{5/2} d^2} c \text{ Cosh}\left[\frac{2 b c}{d}\right] \left( 3 d^{3/2} \sqrt{\pi} \text{ Erf}\left[\frac{\sqrt{2} \sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right] + 3 d^{3/2} \sqrt{\pi} \text{ Erfi}\left[\frac{\sqrt{2} \sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right] + \right.$$

$$\left. 4 \sqrt{2} \sqrt{b} \sqrt{c+d x} \left( -3 d \text{ Cosh}\left[\frac{2 b (c+d x)}{d}\right] + 4 b (c+d x) \text{ Sinh}\left[\frac{2 b (c+d x)}{d}\right] \right) \right) -$$

$$\left( (c+d x)^{3/2} \text{ Sinh}\left[\frac{2 b c}{d}\right] \left( -15 d^2 \sqrt{\pi} \text{ Erf}\left[\sqrt{2} \sqrt{\frac{b (c+d x)}{d}}\right] - 15 d^2 \sqrt{\pi} \text{ Erfi}\left[\sqrt{2} \sqrt{\frac{b (c+d x)}{d}}\right] + 4 \sqrt{2} \sqrt{\frac{b (c+d x)}{d}} \right. \right.$$

$$\left. \left. \left( \left(15 d^2 + 16 b^2 (c+d x)^2\right) \text{Cosh}\left[\frac{2 b (c+d x)}{d}\right] - 20 b d (c+d x) \text{ Sinh}\left[\frac{2 b (c+d x)}{d}\right] \right) \right) \right) \Big/ \left( 128 \sqrt{2} b^2 d^3 \left(\frac{b (c+d x)}{d}\right)^{3/2} \right) +$$

$$\frac{1}{128 \sqrt{2} b^{7/2} d^2} \text{Cosh}\left[\frac{2 b c}{d}\right] \left( 15 d^{5/2} \sqrt{\pi} \text{ Erf}\left[\frac{\sqrt{2} \sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right] - 15 d^{5/2} \sqrt{\pi} \text{ Erfi}\left[\frac{\sqrt{2} \sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right] + \right.$$

$$4\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}\,\left(-20\,b\,d\,(c+d\,x)\,\mathrm{Cosh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]+\left(15\,d^2+16\,b^2\,(c+d\,x)^2\right)\,\mathrm{Sinh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]\right)\Bigg)\Bigg)+$$

$$d^2\,\mathrm{Cosh}[a]\,\mathrm{Sinh}[a]\,\Bigg(\frac{2\,c^2\,\mathrm{Cosh}\left[\frac{2\,b\,c}{d}\right]\left(\frac{d\,\sqrt{c+d\,x}\,\mathrm{Cosh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]}{4\,b}-\frac{d^{3/2}\,\sqrt{\pi}\,\left(\mathrm{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+\mathrm{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]\right)}{16\,\sqrt{2}\,b^{3/2}}\right)}{d^3}-$$

$$\frac{2\,c^2\,\mathrm{Sinh}\left[\frac{2\,b\,c}{d}\right]\left(-\frac{d^{3/2}\,\sqrt{\pi}\,\left(-\mathrm{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+\mathrm{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]\right)}{16\,\sqrt{2}\,b^{3/2}}+\frac{d\,\sqrt{c+d\,x}\,\mathrm{Sinh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]}{4\,b}\right)}{d^3}+\frac{1}{16\,\sqrt{2}\,b^{5/2}\,d^2}$$

$$c\,\mathrm{Cosh}\left[\frac{2\,b\,c}{d}\right]\,\Bigg(3\,d^{3/2}\,\sqrt{\pi}\,\mathrm{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]-3\,d^{3/2}\,\sqrt{\pi}\,\mathrm{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+$$

$$4\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}\,\left(-4\,b\,(c+d\,x)\,\mathrm{Cosh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]+3\,d\,\mathrm{Sinh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]\right)\Bigg)+$$

$$\frac{1}{16\,\sqrt{2}\,b^{5/2}\,d^2}c\,\mathrm{Sinh}\left[\frac{2\,b\,c}{d}\right]\,\Bigg(3\,d^{3/2}\,\sqrt{\pi}\,\mathrm{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+3\,d^{3/2}\,\sqrt{\pi}\,\mathrm{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+$$

$$4\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}\,\left(-3\,d\,\mathrm{Cosh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]+4\,b\,(c+d\,x)\,\mathrm{Sinh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]\right)\Bigg)+$$

$$\frac{1}{128\,\sqrt{2}\,b^{7/2}\,d^2}\mathrm{Cosh}\left[\frac{2\,b\,c}{d}\right]\,\Bigg(-15\,d^{5/2}\,\sqrt{\pi}\,\mathrm{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]-15\,d^{5/2}\,\sqrt{\pi}\,\mathrm{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+$$

$$4\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}\,\left(\left(15\,d^2+16\,b^2\,(c+d\,x)^2\right)\,\mathrm{Cosh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]-20\,b\,d\,(c+d\,x)\,\mathrm{Sinh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]\right)\Bigg)-$$

$$\frac{1}{128\,\sqrt{2}\,b^{7/2}\,d^2}\mathrm{Sinh}\left[\frac{2\,b\,c}{d}\right]\,\Bigg(15\,d^{5/2}\,\sqrt{\pi}\,\mathrm{Erf}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]-15\,d^{5/2}\,\sqrt{\pi}\,\mathrm{Erfi}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]+$$

$$4\sqrt{2}\ \sqrt{b}\ \sqrt{c+d\,x}\ \left(-20\,b\,d\,(c+d\,x)\,\text{Cosh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]+\left(15\,d^2+16\,b^2\,(c+d\,x)^2\right)\,\text{Sinh}\left[\frac{2\,b\,(c+d\,x)}{d}\right]\right)\Bigg)$$

## Problem 61: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Cosh}[a+b\,x]^3}{(c+d\,x)^{5/2}}\,dx$$

Optimal (type 4, 277 leaves, 18 steps) :

$$-\frac{2\,\text{Cosh}[a+b\,x]^3}{3\,d\,(c+d\,x)^{3/2}}+\frac{b^{3/2}\,e^{-a+\frac{b\,c}{d}}\,\sqrt{\pi}\,\text{Erf}\left[\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]}{2\,d^{5/2}}+\frac{b^{3/2}\,e^{-3\,a+\frac{3\,b\,c}{d}}\,\sqrt{3\,\pi}\,\text{Erf}\left[\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]}{2\,d^{5/2}}+$$
$$\frac{b^{3/2}\,e^{a-\frac{b\,c}{d}}\,\sqrt{\pi}\,\text{Erfi}\left[\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]}{2\,d^{5/2}}+\frac{b^{3/2}\,e^{3\,a-\frac{3\,b\,c}{d}}\,\sqrt{3\,\pi}\,\text{Erfi}\left[\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\right]}{2\,d^{5/2}}-\frac{4\,b\,\text{Cosh}[a+b\,x]^2\,\text{Sinh}[a+b\,x]}{d^2\,\sqrt{c+d\,x}}$$

Result (type 4, 716 leaves) :

$$\frac{1}{6\,d^{5/2}\,(c+d\,x)^{3/2}}\Bigg(-3\,d^{3/2}\,\text{Cosh}[a+b\,x]-d^{3/2}\,\text{Cosh}[3\,(a+b\,x)]+$$
$$3\,b^{3/2}\,c\,\sqrt{\pi}\,\sqrt{c+d\,x}\,\text{Cosh}\Big[a-\frac{b\,c}{d}\Big]\,\text{Erfi}\Big[\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]+3\,b^{3/2}\,d\,\sqrt{\pi}\,x\,\sqrt{c+d\,x}\,\text{Cosh}\Big[a-\frac{b\,c}{d}\Big]\,\text{Erfi}\Big[\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]+$$
$$3\,b^{3/2}\,c\,\sqrt{3\,\pi}\,\sqrt{c+d\,x}\,\text{Cosh}\Big[3\,a-\frac{3\,b\,c}{d}\Big]\,\text{Erfi}\Big[\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]+3\,b^{3/2}\,d\,\sqrt{3\,\pi}\,x\,\sqrt{c+d\,x}\,\text{Cosh}\Big[3\,a-\frac{3\,b\,c}{d}\Big]\,\text{Erfi}\Big[\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]+$$
$$3\,b^{3/2}\,\sqrt{3\,\pi}\,(c+d\,x)^{3/2}\,\text{Erf}\Big[\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]\Big(\text{Cosh}\Big[3\,a-\frac{3\,b\,c}{d}\Big]-\text{Sinh}\Big[3\,a-\frac{3\,b\,c}{d}\Big]\Big)+$$
$$3\,b^{3/2}\,c\,\sqrt{3\,\pi}\,\sqrt{c+d\,x}\,\text{Erfi}\Big[\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]\,\text{Sinh}\Big[3\,a-\frac{3\,b\,c}{d}\Big]+3\,b^{3/2}\,d\,\sqrt{3\,\pi}\,x\,\sqrt{c+d\,x}\,\text{Erfi}\Big[\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]\,\text{Sinh}\Big[3\,a-\frac{3\,b\,c}{d}\Big]+$$
$$3\,b^{3/2}\,\sqrt{\pi}\,(c+d\,x)^{3/2}\,\text{Erf}\Big[\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]\Big(\text{Cosh}\Big[a-\frac{b\,c}{d}\Big]-\text{Sinh}\Big[a-\frac{b\,c}{d}\Big]\Big)+$$
$$3\,b^{3/2}\,c\,\sqrt{\pi}\,\sqrt{c+d\,x}\,\text{Erfi}\Big[\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]\,\text{Sinh}\Big[a-\frac{b\,c}{d}\Big]+3\,b^{3/2}\,d\,\sqrt{\pi}\,x\,\sqrt{c+d\,x}\,\text{Erfi}\Big[\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]\,\text{Sinh}\Big[a-\frac{b\,c}{d}\Big]-$$
$$6\,b\,c\,\sqrt{d}\,\text{Sinh}[a+b\,x]-6\,b\,d^{3/2}\,x\,\text{Sinh}[a+b\,x]-6\,b\,c\,\sqrt{d}\,\text{Sinh}[3\,(a+b\,x)]-6\,b\,d^{3/2}\,x\,\text{Sinh}[3\,(a+b\,x)]\Bigg)$$

## Problem 62: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Cosh}[a+b\,x]^3}{(c+d\,x)^{7/2}}\,dx$$

Optimal (type 4, 331 leaves, 19 steps):

$$\frac{16\,b^2\,\text{Cosh}[a+b\,x]}{5\,d^3\,\sqrt{c+d\,x}}-\frac{2\,\text{Cosh}[a+b\,x]^3}{5\,d\,(c+d\,x)^{5/2}}-\frac{24\,b^2\,\text{Cosh}[a+b\,x]^3}{5\,d^3\,\sqrt{c+d\,x}}-\frac{b^{5/2}\,e^{-a+\frac{b\,c}{d}}\,\sqrt{\pi}\,\text{Erf}\Big[\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]}{5\,d^{7/2}}-\frac{3\,b^{5/2}\,e^{-3\,a+\frac{3\,b\,c}{d}}\,\sqrt{3\,\pi}\,\text{Erf}\Big[\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]}{5\,d^{7/2}}+$$
$$\frac{b^{5/2}\,e^{a-\frac{b\,c}{d}}\,\sqrt{\pi}\,\text{Erfi}\Big[\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]}{5\,d^{7/2}}+\frac{3\,b^{5/2}\,e^{3\,a-\frac{3\,b\,c}{d}}\,\sqrt{3\,\pi}\,\text{Erfi}\Big[\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]}{5\,d^{7/2}}-\frac{4\,b\,\text{Cosh}[a+b\,x]^2\,\text{Sinh}[a+b\,x]}{5\,d^2\,(c+d\,x)^{3/2}}$$

Result (type 4, 680 leaves):

$$-\frac{1}{10\,d^{7/2}\,(c+d\,x)^{5/2}}\Big(4\,b^2\,c^2\,\sqrt{d}\,\text{Cosh}[a+b\,x]+3\,d^{5/2}\,\text{Cosh}[a+b\,x]+8\,b^2\,c\,d^{3/2}\,x\,\text{Cosh}[a+b\,x]+4\,b^2\,d^{5/2}\,x^2\,\text{Cosh}[a+b\,x]+12\,b^2\,c^2\,\sqrt{d}\,\text{Cosh}[3\,(a+b\,x)]+$$

$$d^{5/2}\,\text{Cosh}[3\,(a+b\,x)]+24\,b^2\,c\,d^{3/2}\,x\,\text{Cosh}[3\,(a+b\,x)]+12\,b^2\,d^{5/2}\,x^2\,\text{Cosh}[3\,(a+b\,x)]+2\,b^{5/2}\,\sqrt{\pi}\,(c+d\,x)^{5/2}\,\text{Cosh}\Big[a-\frac{b\,c}{d}\Big]\,\text{Erf}\Big[\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]+$$

$$6\,b^{5/2}\,\sqrt{3\,\pi}\,(c+d\,x)^{5/2}\,\text{Cosh}\Big[3\,a-\frac{3\,b\,c}{d}\Big]\,\text{Erf}\Big[\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]-2\,b^{5/2}\,\sqrt{\pi}\,(c+d\,x)^{5/2}\,\text{Cosh}\Big[a-\frac{b\,c}{d}\Big]\,\text{Erfi}\Big[\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]-$$

$$6\,b^{5/2}\,\sqrt{3\,\pi}\,(c+d\,x)^{5/2}\,\text{Cosh}\Big[3\,a-\frac{3\,b\,c}{d}\Big]\,\text{Erfi}\Big[\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]-6\,b^{5/2}\,\sqrt{3\,\pi}\,(c+d\,x)^{5/2}\,\text{Erf}\Big[\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]\,\text{Sinh}\Big[3\,a-\frac{3\,b\,c}{d}\Big]-$$

$$6\,b^{5/2}\,\sqrt{3\,\pi}\,(c+d\,x)^{5/2}\,\text{Erfi}\Big[\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]\,\text{Sinh}\Big[3\,a-\frac{3\,b\,c}{d}\Big]-2\,b^{5/2}\,\sqrt{\pi}\,(c+d\,x)^{5/2}\,\text{Erf}\Big[\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]\,\text{Sinh}\Big[a-\frac{b\,c}{d}\Big]-$$

$$2\,b^{5/2}\,\sqrt{\pi}\,(c+d\,x)^{5/2}\,\text{Erfi}\Big[\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{d}}\Big]\,\text{Sinh}\Big[a-\frac{b\,c}{d}\Big]+2\,b\,c\,d^{3/2}\,\text{Sinh}[a+b\,x]+$$

$$2\,b\,d^{5/2}\,x\,\text{Sinh}[a+b\,x]+2\,b\,c\,d^{3/2}\,\text{Sinh}[3\,(a+b\,x)]+2\,b\,d^{5/2}\,x\,\text{Sinh}[3\,(a+b\,x)]\Big)$$

## Problem 71: Result more than twice size of optimal antiderivative.

$$\int\left(\frac{x}{\text{Cosh}[x]^{3/2}}+x\,\sqrt{\text{Cosh}[x]}\right)dx$$

Optimal (type 3, 20 leaves, 2 steps):

$$-4\,\sqrt{\text{Cosh}[x]}+\frac{2\,x\,\text{Sinh}[x]}{\sqrt{\text{Cosh}[x]}}$$

Result (type 3, 46 leaves):

$$\frac{2\,\text{Sinh}[x]\left(x-\frac{2\,\text{Cosh}[x]\,\text{Sinh}[x]\,\sqrt{\text{Tanh}\left[\frac{x}{2}\right]^2}}{(-1+\text{Cosh}[x])^{3/2}\,\sqrt{1+\text{Cosh}[x]}}\right)}{\sqrt{\text{Cosh}[x]}}$$

## Problem 74: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(\frac{x^2}{\text{Cosh}[x]^{3/2}} + x^2 \sqrt{\text{Cosh}[x]}\right) dx$$

Optimal (type 4, 36 leaves, 3 steps):

$$-8 x \sqrt{\text{Cosh}[x]} - 16 i \,\text{EllipticE}\left[\frac{i x}{2}, 2\right] + \frac{2 x^2 \,\text{Sinh}[x]}{\sqrt{\text{Cosh}[x]}}$$

Result (type 5, 76 leaves):

$$\frac{1}{1 + e^{2 x}} 4 \sqrt{\text{Cosh}[x]} \,(\text{Cosh}[x] + \text{Sinh}[x]) \left(-4 (-2 + x) \,\text{Cosh}[x] + x^2 \,\text{Sinh}[x] + 8 \,\text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 x}\right] (-\text{Cosh}[x] + \text{Sinh}[x]) \sqrt{1 + \text{Cosh}[2 x] + \text{Sinh}[2 x]}\right)$$

## Problem 76: Attempted integration timed out after 120 seconds.

$$\int (c + d x)^m \,\text{Cosh}[a + b x]^3 \, dx$$

Optimal (type 4, 237 leaves, 8 steps):

$$\frac{3^{-1-m} e^{3 a-\frac{3 b c}{d}} (c + d x)^m \left(-\frac{b (c+d x)}{d}\right)^{-m} \text{Gamma}\left[1 + m, -\frac{3 b (c+d x)}{d}\right]}{8 b} + \frac{3 e^{a-\frac{b c}{d}} (c + d x)^m \left(-\frac{b (c+d x)}{d}\right)^{-m} \text{Gamma}\left[1 + m, -\frac{b (c+d x)}{d}\right]}{8 b} - \frac{3 e^{-a+\frac{b c}{d}} (c + d x)^m \left(\frac{b (c+d x)}{d}\right)^{-m} \text{Gamma}\left[1 + m, \frac{b (c+d x)}{d}\right]}{8 b} - \frac{3^{-1-m} e^{-3 a+\frac{3 b c}{d}} (c + d x)^m \left(\frac{b (c+d x)}{d}\right)^{-m} \text{Gamma}\left[1 + m, \frac{3 b (c+d x)}{d}\right]}{8 b}$$

Result (type 1, 1 leaves):

???

## Problem 112: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^2}{a + a \,\text{Cosh}[e + f x]} \, dx$$

Optimal (type 4, 88 leaves, 6 steps):

$$\frac{(c + d x)^2}{a f} - \frac{4 d (c + d x) \,\text{Log}\left[1 + e^{e+f x}\right]}{a f^2} - \frac{4 d^2 \,\text{PolyLog}\left[2, -e^{e+f x}\right]}{a f^3} + \frac{(c + d x)^2 \,\text{Tanh}\left[\frac{e}{2} + \frac{f x}{2}\right]}{a f}$$

Result (type 4, 472 leaves) :

$$-\frac{8\,c\,d\,\text{Cosh}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^2\,\text{Sech}\left[\frac{e}{2}\right]\,\left(\text{Cosh}\left[\frac{e}{2}\right]\,\text{Log}\left[\text{Cosh}\left[\frac{e}{2}\right]\,\text{Cosh}\left[\frac{f\,x}{2}\right]+\text{Sinh}\left[\frac{e}{2}\right]\,\text{Sinh}\left[\frac{f\,x}{2}\right]\right]-\frac{1}{2}\,f\,x\,\text{Sinh}\left[\frac{e}{2}\right]\right)}{f^2\,\left(a+a\,\text{Cosh}[e+f\,x]\right)\,\left(\text{Cosh}\left[\frac{e}{2}\right]^2-\text{Sinh}\left[\frac{e}{2}\right]^2\right)}+$$

$$\left(8\,d^2\,\text{Cosh}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^2\,\text{Csch}\left[\frac{e}{2}\right]\,\left(-\frac{1}{4}\,e^{-\text{ArcTanh}\left[\text{Coth}\left[\frac{e}{2}\right]\right]}\,f^2\,x^2+\frac{1}{\sqrt{1-\text{Coth}\left[\frac{e}{2}\right]^2}}\right.\right.$$

$$i\,\text{Coth}\left[\frac{e}{2}\right]\,\left(-\frac{1}{2}\,f\,x\,\left(-\pi+2\,i\,\text{ArcTanh}\left[\text{Coth}\left[\frac{e}{2}\right]\right]\right)-\pi\,\text{Log}\left[1+e^{f\,x}\right]-2\,\left(\frac{i\,f\,x}{2}+i\,\text{ArcTanh}\left[\text{Coth}\left[\frac{e}{2}\right]\right]\right)\,\text{Log}\left[1-e^{2\,i\,\left(\frac{i\,f\,x}{2}+i\,\text{ArcTanh}\left[\text{Coth}\left[\frac{e}{2}\right]\right]\right)}\right]+\pi\right.$$

$$\left.\left.\left.\text{Log}\left[\text{Cosh}\left[\frac{f\,x}{2}\right]\right]+2\,i\,\text{ArcTanh}\left[\text{Coth}\left[\frac{e}{2}\right]\right]\,\text{Log}\left[i\,\text{Sinh}\left[\frac{f\,x}{2}+\text{ArcTanh}\left[\text{Coth}\left[\frac{e}{2}\right]\right]\right]\right]+i\,\text{PolyLog}\left[2,\,e^{2\,i\,\left(\frac{i\,f\,x}{2}+i\,\text{ArcTanh}\left[\text{Coth}\left[\frac{e}{2}\right]\right]\right)}\right]\right)\right)\,\text{Sech}\left[\frac{e}{2}\right]\right)\Bigg/$$

$$\left(f^3\,\left(a+a\,\text{Cosh}[e+f\,x]\right)\,\sqrt{\text{Csch}\left[\frac{e}{2}\right]^2\,\left(-\text{Cosh}\left[\frac{e}{2}\right]^2+\text{Sinh}\left[\frac{e}{2}\right]^2\right)}\right)+\frac{2\,\text{Cosh}\left[\frac{e}{2}+\frac{f\,x}{2}\right]\,\text{Sech}\left[\frac{e}{2}\right]\,\left(c^2\,\text{Sinh}\left[\frac{f\,x}{2}\right]+2\,c\,d\,x\,\text{Sinh}\left[\frac{f\,x}{2}\right]+d^2\,x^2\,\text{Sinh}\left[\frac{f\,x}{2}\right]\right)}{f\,\left(a+a\,\text{Cosh}[e+f\,x]\right)}$$

## Problem 117: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{(c+d\,x)^2}{\left(a+a\,\text{Cosh}[e+f\,x]\right)^2}\,dx$$

Optimal (type 4, 200 leaves, 9 steps) :

$$\frac{(c+d\,x)^2}{3\,a^2\,f}-\frac{4\,d\,(c+d\,x)\,\text{Log}\left[1+e^{e+f\,x}\right]}{3\,a^2\,f^2}-\frac{4\,d^2\,\text{PolyLog}\left[2,\,-e^{e+f\,x}\right]}{3\,a^2\,f^3}+$$

$$\frac{d\,(c+d\,x)\,\text{Sech}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^2}{3\,a^2\,f^2}-\frac{2\,d^2\,\text{Tanh}\left[\frac{e}{2}+\frac{f\,x}{2}\right]}{3\,a^2\,f^3}+\frac{(c+d\,x)^2\,\text{Tanh}\left[\frac{e}{2}+\frac{f\,x}{2}\right]}{3\,a^2\,f}+\frac{(c+d\,x)^2\,\text{Sech}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^2\,\text{Tanh}\left[\frac{e}{2}+\frac{f\,x}{2}\right]}{6\,a^2\,f}$$

Result (type 4, 637 leaves) :

$$-\frac{16\,c\,d\,\text{Cosh}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^4\,\text{Sech}\left[\frac{e}{2}\right]\left(\text{Cosh}\left[\frac{e}{2}\right]\,\text{Log}\left[\text{Cosh}\left[\frac{e}{2}\right]\,\text{Cosh}\left[\frac{f\,x}{2}\right]+\text{Sinh}\left[\frac{e}{2}\right]\,\text{Sinh}\left[\frac{f\,x}{2}\right]\right]-\frac{1}{2}\,f\,x\,\text{Sinh}\left[\frac{e}{2}\right]\right)}{3\,f^2\,\left(a+a\,\text{Cosh}[e+f\,x]\right)^2\,\left(\text{Cosh}\left[\frac{e}{2}\right]^2-\text{Sinh}\left[\frac{e}{2}\right]^2\right)}+$$

$$\left(16\,d^2\,\text{Cosh}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^4\,\text{Csch}\left[\frac{e}{2}\right]\left(-\frac{1}{4}\,e^{-\text{ArcTanh}\left[\text{Coth}\left[\frac{e}{2}\right]\right]}\,f^2\,x^2+\frac{1}{\sqrt{1-\text{Coth}\left[\frac{e}{2}\right]^2}}\right.\right.$$

$$i\,\text{Coth}\left[\frac{e}{2}\right]\left(-\frac{1}{2}\,f\,x\left(-\pi+2\,i\,\text{ArcTanh}\left[\text{Coth}\left[\frac{e}{2}\right]\right]\right)-\pi\,\text{Log}\left[1+e^{f\,x}\right]-2\left(\frac{i\,f\,x}{2}+i\,\text{ArcTanh}\left[\text{Coth}\left[\frac{e}{2}\right]\right]\right)\text{Log}\left[1-e^{2\,i\,\left(\frac{i\,f\,x}{2}+i\,\text{ArcTanh}\left[\text{Coth}\left[\frac{e}{2}\right]\right]\right)}\right]+\pi\right.$$

$$\left.\left.\left.\text{Log}\left[\text{Cosh}\left[\frac{f\,x}{2}\right]\right]+2\,i\,\text{ArcTanh}\left[\text{Coth}\left[\frac{e}{2}\right]\right]\,\text{Log}\left[i\,\text{Sinh}\left[\frac{f\,x}{2}+\text{ArcTanh}\left[\text{Coth}\left[\frac{e}{2}\right]\right]\right]\right]+i\,\text{PolyLog}\left[2,\,e^{2\,i\,\left(\frac{i\,f\,x}{2}+i\,\text{ArcTanh}\left[\text{Coth}\left[\frac{e}{2}\right]\right]\right)}\right]\right)\right)\,\text{Sech}\left[\frac{e}{2}\right]\right)\Big/$$

$$\left(3\,f^3\,\left(a+a\,\text{Cosh}[e+f\,x]\right)^2\sqrt{\text{Csch}\left[\frac{e}{2}\right]^2\left(-\text{Cosh}\left[\frac{e}{2}\right]^2+\text{Sinh}\left[\frac{e}{2}\right]^2\right)}\right)+\frac{1}{3\,f^3\,\left(a+a\,\text{Cosh}[e+f\,x]\right)^2}\,\text{Cosh}\left[\frac{e}{2}+\frac{f\,x}{2}\right]\,\text{Sech}\left[\frac{e}{2}\right]$$

$$\left(2\,c\,d\,f\,\text{Cosh}\left[\frac{f\,x}{2}\right]+2\,d^2\,f\,x\,\text{Cosh}\left[\frac{f\,x}{2}\right]+2\,c\,d\,f\,\text{Cosh}\left[e+\frac{f\,x}{2}\right]+2\,d^2\,f\,x\,\text{Cosh}\left[e+\frac{f\,x}{2}\right]-4\,d^2\,\text{Sinh}\left[\frac{f\,x}{2}\right]+3\,c^2\,f^2\,\text{Sinh}\left[\frac{f\,x}{2}\right]+6\,c\,d\,f^2\,x\,\text{Sinh}\left[\frac{f\,x}{2}\right]+\right.$$

$$\left.3\,d^2\,f^2\,x^2\,\text{Sinh}\left[\frac{f\,x}{2}\right]+2\,d^2\,\text{Sinh}\left[e+\frac{f\,x}{2}\right]-2\,d^2\,\text{Sinh}\left[e+\frac{3\,f\,x}{2}\right]+c^2\,f^2\,\text{Sinh}\left[e+\frac{3\,f\,x}{2}\right]+2\,c\,d\,f^2\,x\,\text{Sinh}\left[e+\frac{3\,f\,x}{2}\right]+d^2\,f^2\,x^2\,\text{Sinh}\left[e+\frac{3\,f\,x}{2}\right]\right)$$

## Problem 144: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{\left(a+a\,\text{Cosh}[x]\right)^{3/2}}\,dx$$

Optimal (type 4, 402 leaves, 16 steps):

$$\frac{3\,x^2}{a\,\sqrt{a+a\,\text{Cosh}[x]}}-\frac{24\,x\,\text{ArcTan}\left[e^{x/2}\right]\,\text{Cosh}\left[\frac{x}{2}\right]}{a\,\sqrt{a+a\,\text{Cosh}[x]}}+\frac{x^3\,\text{ArcTan}\left[e^{x/2}\right]\,\text{Cosh}\left[\frac{x}{2}\right]}{a\,\sqrt{a+a\,\text{Cosh}[x]}}+\frac{24\,i\,\text{Cosh}\left[\frac{x}{2}\right]\,\text{PolyLog}\left[2,\,-i\,e^{x/2}\right]}{a\,\sqrt{a+a\,\text{Cosh}[x]}}-$$

$$\frac{3\,i\,x^2\,\text{Cosh}\left[\frac{x}{2}\right]\,\text{PolyLog}\left[2,\,-i\,e^{x/2}\right]}{a\,\sqrt{a+a\,\text{Cosh}[x]}}-\frac{24\,i\,\text{Cosh}\left[\frac{x}{2}\right]\,\text{PolyLog}\left[2,\,i\,e^{x/2}\right]}{a\,\sqrt{a+a\,\text{Cosh}[x]}}+\frac{3\,i\,x^2\,\text{Cosh}\left[\frac{x}{2}\right]\,\text{PolyLog}\left[2,\,i\,e^{x/2}\right]}{a\,\sqrt{a+a\,\text{Cosh}[x]}}+\frac{12\,i\,x\,\text{Cosh}\left[\frac{x}{2}\right]\,\text{PolyLog}\left[3,\,-i\,e^{x/2}\right]}{a\,\sqrt{a+a\,\text{Cosh}[x]}}-$$

$$\frac{12\,i\,x\,\text{Cosh}\left[\frac{x}{2}\right]\,\text{PolyLog}\left[3,\,i\,e^{x/2}\right]}{a\,\sqrt{a+a\,\text{Cosh}[x]}}-\frac{24\,i\,\text{Cosh}\left[\frac{x}{2}\right]\,\text{PolyLog}\left[4,\,-i\,e^{x/2}\right]}{a\,\sqrt{a+a\,\text{Cosh}[x]}}+\frac{24\,i\,\text{Cosh}\left[\frac{x}{2}\right]\,\text{PolyLog}\left[4,\,i\,e^{x/2}\right]}{a\,\sqrt{a+a\,\text{Cosh}[x]}}+\frac{x^3\,\text{Tanh}\left[\frac{x}{2}\right]}{2\,a\,\sqrt{a+a\,\text{Cosh}[x]}}$$

Result (type 4, 1323 leaves):

$$\frac{6 x^2 \text{Cosh}\left[\frac{x}{2}\right]^2}{\left(a \left(1+\text{Cosh}[x]\right)\right)^{3/2}}-\frac{1}{\left(a \left(1+\text{Cosh}[x]\right)\right)^{3/2}}$$

$$48 \text{Cosh}\left[\frac{x}{2}\right]^3 \left(-\frac{1}{2} i x \left(\text{Log}\left[1-i e^{-x/2}\right]-\text{Log}\left[1+i e^{-x/2}\right]\right)-i \left(\text{PolyLog}\left[2,-i e^{-x/2}\right]-\text{PolyLog}\left[2,i e^{-x/2}\right]\right)\right)+\frac{1}{\left(a \left(1+\text{Cosh}[x]\right)\right)^{3/2}} 8 \text{Cosh}\left[\frac{x}{2}\right]^3$$

$$\left(\frac{1}{8} \pi^3 \text{Log}\left[\text{Cot}\left[\frac{1}{2} \left(\frac{\pi}{2}-\frac{i x}{2}\right)\right]\right]+\frac{3}{4} \pi^2 \left(\left(\frac{\pi}{2}-\frac{i x}{2}\right) \left(\text{Log}\left[1-e^{i \left(\frac{\pi}{2}-\frac{i x}{2}\right)}\right]-\text{Log}\left[1+e^{i \left(\frac{\pi}{2}-\frac{i x}{2}\right)}\right]\right)+i \left(\text{PolyLog}\left[2,-e^{i \left(\frac{\pi}{2}-\frac{i x}{2}\right)}\right]-\text{PolyLog}\left[2,e^{i \left(\frac{\pi}{2}-\frac{i x}{2}\right)}\right]\right)\right)-\right.$$

$$\frac{3}{2} \pi \left(\left(\frac{\pi}{2}-\frac{i x}{2}\right)^2 \left(\text{Log}\left[1-e^{i \left(\frac{\pi}{2}-\frac{i x}{2}\right)}\right]-\text{Log}\left[1+e^{i \left(\frac{\pi}{2}-\frac{i x}{2}\right)}\right]\right)+2 i \left(\frac{\pi}{2}-\frac{i x}{2}\right) \left(\text{PolyLog}\left[2,-e^{i \left(\frac{\pi}{2}-\frac{i x}{2}\right)}\right]-\text{PolyLog}\left[2,e^{i \left(\frac{\pi}{2}-\frac{i x}{2}\right)}\right]\right)+\right.$$

$$\left.2 \left(-\text{PolyLog}\left[3,-e^{i \left(\frac{\pi}{2}-\frac{i x}{2}\right)}\right]+\text{PolyLog}\left[3,e^{i \left(\frac{\pi}{2}-\frac{i x}{2}\right)}\right]\right)\right)+8 \left(\frac{1}{4} i \left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right)^4+\frac{1}{64} i \left(\frac{\pi}{2}-\frac{i x}{2}\right)^4-\right.$$

$$\frac{1}{8} \pi^3 \left(i \left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right)-\text{Log}\left[1+e^{2 i \left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right)}\right]\right)-\left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right)^3 \text{Log}\left[1+e^{2 i \left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right)}\right]-\frac{1}{8} \left(\frac{\pi}{2}-\frac{i x}{2}\right)^3 \text{Log}\left[1+e^{i \left(\frac{\pi}{2}-\frac{i x}{2}\right)}\right]+$$

$$\frac{3}{4} \pi^2 \left(\frac{1}{2} i \left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right)^2-\left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right) \text{Log}\left[1+e^{2 i \left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right)}\right]+\frac{1}{2} i \text{PolyLog}\left[2,-e^{2 i \left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right)}\right]\right)+$$

$$\frac{3}{2} i \left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right)^2 \text{PolyLog}\left[2,-e^{2 i \left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right)}\right]+\frac{3}{8} i \left(\frac{\pi}{2}-\frac{i x}{2}\right)^2 \text{PolyLog}\left[2,-e^{i \left(\frac{\pi}{2}-\frac{i x}{2}\right)}\right]-$$

$$\frac{3}{2} \pi \left(\frac{1}{3} i \left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right)^3-\left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right)^2 \text{Log}\left[1+e^{2 i \left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right)}\right]+i \left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right) \text{PolyLog}\left[2,-e^{2 i \left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right)}\right]-\right.$$

$$\left.\frac{1}{2} \text{PolyLog}\left[3,-e^{2 i \left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right)}\right]\right)-\frac{3}{2} \left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right) \text{PolyLog}\left[3,-e^{2 i \left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right)}\right]-$$

$$\left.\left.\frac{3}{4} \left(\frac{\pi}{2}-\frac{i x}{2}\right) \text{PolyLog}\left[3,-e^{i \left(\frac{\pi}{2}-\frac{i x}{2}\right)}\right]-\frac{3}{4} i \text{PolyLog}\left[4,-e^{2 i \left(\frac{\pi}{2}+\frac{1}{2} \left(-\frac{\pi}{2}+\frac{i x}{2}\right)\right)}\right]-\frac{3}{4} i \text{PolyLog}\left[4,-e^{i \left(\frac{\pi}{2}-\frac{i x}{2}\right)}\right]\right)\right)+\frac{x^3 \text{Cosh}\left[\frac{x}{2}\right] \text{Sinh}\left[\frac{x}{2}\right]}{\left(a \left(1+\text{Cosh}[x]\right)\right)^{3/2}}$$

## Problem 168: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+d x)^3}{a+b \text{Cosh}[e+f x]} \, dx$$

Optimal (type 4, 436 leaves, 12 steps):

$$\frac{(c+d x)^3 \text{Log}\left[1+\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2} f}-\frac{(c+d x)^3 \text{Log}\left[1+\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2} f}+\frac{3 d (c+d x)^2 \text{PolyLog}\left[2,-\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2} f^2}-\frac{3 d (c+d x)^2 \text{PolyLog}\left[2,-\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2} f^2}-$$

$$\frac{6 d^2 (c+d x) \text{PolyLog}\left[3,-\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2} f^3}+\frac{6 d^2 (c+d x) \text{PolyLog}\left[3,-\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2} f^3}+\frac{6 d^3 \text{PolyLog}\left[4,-\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2} f^4}-\frac{6 d^3 \text{PolyLog}\left[4,-\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2} f^4}$$

Result (type 4, 1031 leaves) :

$$\frac{1}{\sqrt{-a^2+b^2}\sqrt{(a^2-b^2)\,e^{2e}}\,f^4}\Bigg(2c^3\sqrt{(a^2-b^2)\,e^{2e}}\,f^3\,\text{ArcTan}\Big[\frac{a+b\,e^{e+fx}}{\sqrt{-a^2+b^2}}\Big]+3\sqrt{-a^2+b^2}\,c^2 d\,e^e f^3 x\,\text{Log}\Big[1+\frac{b\,e^{2e+fx}}{a\,e^e-\sqrt{(a^2-b^2)\,e^{2e}}}\Big]+3\sqrt{-a^2+b^2}\,c\,d^2 e^e f^3 x^2$$

$$\text{Log}\Big[1+\frac{b\,e^{2e+fx}}{a\,e^e-\sqrt{(a^2-b^2)\,e^{2e}}}\Big]+\sqrt{-a^2+b^2}\,d^3 e^e f^3 x^3\,\text{Log}\Big[1+\frac{b\,e^{2e+fx}}{a\,e^e-\sqrt{(a^2-b^2)\,e^{2e}}}\Big]-3\sqrt{-a^2+b^2}\,c^2 d\,e^e f^3 x\,\text{Log}\Big[1+\frac{b\,e^{2e+fx}}{a\,e^e+\sqrt{(a^2-b^2)\,e^{2e}}}\Big]-$$

$$3\sqrt{-a^2+b^2}\,c\,d^2 e^e f^3 x^2\,\text{Log}\Big[1+\frac{b\,e^{2e+fx}}{a\,e^e+\sqrt{(a^2-b^2)\,e^{2e}}}\Big]-\sqrt{-a^2+b^2}\,d^3 e^e f^3 x^3\,\text{Log}\Big[1+\frac{b\,e^{2e+fx}}{a\,e^e+\sqrt{(a^2-b^2)\,e^{2e}}}\Big]+$$

$$3\sqrt{-a^2+b^2}\,d\,e^e f^2 (c+dx)^2\,\text{PolyLog}\Big[2,-\frac{b\,e^{2e+fx}}{a\,e^e-\sqrt{(a^2-b^2)\,e^{2e}}}\Big]-3\sqrt{-a^2+b^2}\,d\,e^e f^2 (c+dx)^2\,\text{PolyLog}\Big[2,-\frac{b\,e^{2e+fx}}{a\,e^e+\sqrt{(a^2-b^2)\,e^{2e}}}\Big]-$$

$$6\sqrt{-a^2+b^2}\,c\,d^2 e^e f\,\text{PolyLog}\Big[3,-\frac{b\,e^{2e+fx}}{a\,e^e-\sqrt{(a^2-b^2)\,e^{2e}}}\Big]-6\sqrt{-a^2+b^2}\,d^3 e^e f x\,\text{PolyLog}\Big[3,-\frac{b\,e^{2e+fx}}{a\,e^e-\sqrt{(a^2-b^2)\,e^{2e}}}\Big]+$$

$$6\sqrt{-a^2+b^2}\,c\,d^2 e^e f\,\text{PolyLog}\Big[3,-\frac{b\,e^{2e+fx}}{a\,e^e+\sqrt{(a^2-b^2)\,e^{2e}}}\Big]+6\sqrt{-a^2+b^2}\,d^3 e^e f x\,\text{PolyLog}\Big[3,-\frac{b\,e^{2e+fx}}{a\,e^e+\sqrt{(a^2-b^2)\,e^{2e}}}\Big]+$$

$$6\sqrt{-a^2+b^2}\,d^3 e^e\,\text{PolyLog}\Big[4,-\frac{b\,e^{2e+fx}}{a\,e^e-\sqrt{(a^2-b^2)\,e^{2e}}}\Big]-6\sqrt{-a^2+b^2}\,d^3 e^e\,\text{PolyLog}\Big[4,-\frac{b\,e^{2e+fx}}{a\,e^e+\sqrt{(a^2-b^2)\,e^{2e}}}\Big]\Bigg)$$

## Problem 173: Attempted integration timed out after 120 seconds.

$$\int\frac{(c+dx)^3}{(a+b\,\text{Cosh}[e+fx])^2}\,dx$$

Optimal (type 4, 823 leaves, 22 steps) :

$$-\frac{(c+d x)^3}{(a^2-b^2) f}+\frac{3 d (c+d x)^2 \text{Log}\left[1+\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^2}+\frac{a (c+d x)^3 \text{Log}\left[1+\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f}+\frac{3 d (c+d x)^2 \text{Log}\left[1+\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^2}-\frac{a (c+d x)^3 \text{Log}\left[1+\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f}+$$

$$\frac{6 d^2 (c+d x) \text{PolyLog}\left[2,-\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^3}+\frac{3 a d (c+d x)^2 \text{PolyLog}\left[2,-\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^2}+\frac{6 d^2 (c+d x) \text{PolyLog}\left[2,-\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^3}-$$

$$\frac{3 a d (c+d x)^2 \text{PolyLog}\left[2,-\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^2}-\frac{6 d^3 \text{PolyLog}\left[3,-\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^4}-\frac{6 a d^2 (c+d x) \text{PolyLog}\left[3,-\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^3}-\frac{6 d^3 \text{PolyLog}\left[3,-\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^4}+$$

$$\frac{6 a d^2 (c+d x) \text{PolyLog}\left[3,-\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^3}+\frac{6 a d^3 \text{PolyLog}\left[4,-\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^4}-\frac{6 a d^3 \text{PolyLog}\left[4,-\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^4}-\frac{b (c+d x)^3 \text{Sinh}[e+f x]}{(a^2-b^2) f (a+b \text{Cosh}[e+f x])}$$

Result (type 1, 1 leaves):

???

## Problem 174: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+d x)^2}{(a+b \text{Cosh}[e+f x])^2} \, dx$$

Optimal (type 4, 593 leaves, 18 steps):

$$-\frac{(c+d x)^2}{(a^2-b^2) f}+\frac{2 d (c+d x) \text{Log}\left[1+\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^2}+\frac{a (c+d x)^2 \text{Log}\left[1+\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f}+\frac{2 d (c+d x) \text{Log}\left[1+\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^2}-$$

$$\frac{a (c+d x)^2 \text{Log}\left[1+\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f}+\frac{2 d^2 \text{PolyLog}\left[2,-\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^3}+\frac{2 a d (c+d x) \text{PolyLog}\left[2,-\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^2}+\frac{2 d^2 \text{PolyLog}\left[2,-\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^3}-$$

$$\frac{2 a d (c+d x) \text{PolyLog}\left[2,-\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^2}-\frac{2 a d^2 \text{PolyLog}\left[3,-\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^3}+\frac{2 a d^2 \text{PolyLog}\left[3,-\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^3}-\frac{b (c+d x)^2 \text{Sinh}[e+f x]}{(a^2-b^2) f (a+b \text{Cosh}[e+f x])}$$

Result (type 4, 6016 leaves):

$$\frac{1}{\left(a^2-b^2\right)\left(1+e^{2e}\right) f} 2 e^{e} \left(-2 c d e^{e} x + 2 c d e^{-e}\left(1+e^{2e}\right) x - d^2 e^{e} x^2 + d^2 e^{-e}\left(1+e^{2e}\right) x^2 + \right.$$

$$\frac{a c^2 e^{-e} \operatorname{ArcTan}\left[\frac{a+b e^{e+f x}}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}} + \frac{a c^2 e^{e} \operatorname{ArcTan}\left[\frac{a+b e^{e+f x}}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}} - \frac{2 a c d e^{-e} \operatorname{ArcTan}\left[\frac{a+b e^{e+f x}}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}\, f} - \frac{2 a c d e^{e} \operatorname{ArcTan}\left[\frac{a+b e^{e+f x}}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}\, f} +$$

$$c d e^{-e}\left(-2 x + \frac{2 a \operatorname{ArcTan}\left[\frac{a+b e^{e+f x}}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}\, f} + \frac{\operatorname{Log}\left[b+2 a e^{e+f x}+b e^{2(e+f x)}\right]}{f}\right) + c d e^{e}\left(-2 x + \frac{2 a \operatorname{ArcTan}\left[\frac{a+b e^{e+f x}}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}\, f} + \frac{\operatorname{Log}\left[b+2 a e^{e+f x}+b e^{2(e+f x)}\right]}{f}\right) -$$

$$2 b d^2 e^{-e}\left(-\frac{\frac{x^2}{2\left(a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right)} - \frac{x \operatorname{Log}\left[1+\frac{b e^{2e+f x}}{a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f} - \frac{\operatorname{PolyLog}\left[2,-\frac{b e^{2e+f x}}{a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f^2}}{\frac{-a e^{-e}-e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}}{b} - \frac{-a e^{-e}+e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}}{b}} + \right.$$

$$\left.\frac{\frac{x^2}{2\left(a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right)} - \frac{x \operatorname{Log}\left[1+\frac{b e^{2e+f x}}{a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f} - \frac{\operatorname{PolyLog}\left[2,-\frac{b e^{2e+f x}}{a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f^2}}{\frac{-a e^{-e}-e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}}{b} - \frac{-a e^{-e}+e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}}{b}}\right) - 2 b d^2 e^{e}$$

$$\left(-\frac{\frac{x^2}{2\left(a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right)} - \frac{x \operatorname{Log}\left[1+\frac{b e^{2e+f x}}{a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f} - \frac{\operatorname{PolyLog}\left[2,-\frac{b e^{2e+f x}}{a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f^2}}{\frac{-a e^{-e}-e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}}{b} - \frac{-a e^{-e}+e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}}{b}} + \frac{\frac{x^2}{2\left(a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right)} - \frac{x \operatorname{Log}\left[1+\frac{b e^{2e+f x}}{a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f} - \frac{\operatorname{PolyLog}\left[2,-\frac{b e^{2e+f x}}{a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f^2}}{\frac{-a e^{-e}-e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}}{b} - \frac{-a e^{-e}+e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}}{b}}\right) -$$

$$2 a d^2 \left(-\left(\left(\left(-a e^{-e}+e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}\right)\left(\frac{x^2}{2\left(a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right)} - \frac{x \operatorname{Log}\left[1+\frac{b e^{2e+f x}}{a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f} - \frac{\operatorname{PolyLog}\left[2,-\frac{b e^{2e+f x}}{a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f^2}\right)\right) \right/$$

$$\left(b\left(\frac{-a\,\mathrm{e}^{-e}-\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}}{b}-\frac{-a\,\mathrm{e}^{-e}+\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}}{b}\right)\right)\Bigg)+$$

$$\left(\left(-a\,\mathrm{e}^{-e}-\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}\right)\left(\frac{x^2}{2\left(a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}\right)}-\frac{x\,\mathrm{Log}\left[1+\frac{b\,\mathrm{e}^{2e+f x}}{a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}}\right]}{\left(a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}\right)f}-\frac{\mathrm{PolyLog}\left[2,-\frac{b\,\mathrm{e}^{2e+f x}}{a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}}\right]}{\left(a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}\right)f^2}\right)\right)\Big/$$

$$\left(b\left(\frac{-a\,\mathrm{e}^{-e}-\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}}{b}-\frac{-a\,\mathrm{e}^{-e}+\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}}{b}\right)\right)\Bigg)+2\,a\,c\,d\,f$$

$$\Bigg(-\Bigg(\Bigg(\left(-a\,\mathrm{e}^{-e}+\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}\right)\left(\frac{x^2}{2\left(a\,\mathrm{e}^{e}-\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}\right)}-\frac{x\,\mathrm{Log}\left[1+\frac{b\,\mathrm{e}^{2e+f x}}{a\,\mathrm{e}^{e}-\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}}\right]}{\left(a\,\mathrm{e}^{e}-\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}\right)f}-\frac{\mathrm{PolyLog}\left[2,-\frac{b\,\mathrm{e}^{2e+f x}}{a\,\mathrm{e}^{e}-\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}}\right]}{\left(a\,\mathrm{e}^{e}-\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}\right)f^2}\right)\Bigg)\Big/$$

$$\left(b\left(\frac{-a\,\mathrm{e}^{-e}-\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}}{b}-\frac{-a\,\mathrm{e}^{-e}+\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}}{b}\right)\right)\Bigg)+$$

$$\left(\left(-a\,\mathrm{e}^{-e}-\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}\right)\left(\frac{x^2}{2\left(a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}\right)}-\frac{x\,\mathrm{Log}\left[1+\frac{b\,\mathrm{e}^{2e+f x}}{a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}}\right]}{\left(a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}\right)f}-\frac{\mathrm{PolyLog}\left[2,-\frac{b\,\mathrm{e}^{2e+f x}}{a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}}\right]}{\left(a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}\right)f^2}\right)\right)\Big/$$

$$\left(b\left(\frac{-a\,\mathrm{e}^{-e}-\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}}{b}-\frac{-a\,\mathrm{e}^{-e}+\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}}{b}\right)\right)\Bigg)-2\,a\,d^2$$

$$\Bigg(-\Bigg(\Bigg(\mathrm{e}^{2e}\left(-a\,\mathrm{e}^{-e}+\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}\right)\left(\frac{x^2}{2\left(a\,\mathrm{e}^{e}-\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}\right)}-\frac{x\,\mathrm{Log}\left[1+\frac{b\,\mathrm{e}^{2e+f x}}{a\,\mathrm{e}^{e}-\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}}\right]}{\left(a\,\mathrm{e}^{e}-\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}\right)f}-\frac{\mathrm{PolyLog}\left[2,-\frac{b\,\mathrm{e}^{2e+f x}}{a\,\mathrm{e}^{e}-\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}}\right]}{\left(a\,\mathrm{e}^{e}-\sqrt{-\left(-a^2+b^2\right)\,\mathrm{e}^{2e}}\right)f^2}\right)\Bigg)\Big/$$

$$\left(b\left(\frac{-a\,\mathrm{e}^{-e}-\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}}{b}-\frac{-a\,\mathrm{e}^{-e}+\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}}{b}\right)\right)\Bigg)+$$

$$\left(e^{2e}\left(-a e^{-e}-e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}\right)\left(\frac{x^2}{2\left(a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right)}-\frac{x\,\text{Log}\left[1+\frac{b e^{2e+fx}}{a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f}-\frac{\text{PolyLog}\left[2,-\frac{b e^{2e+fx}}{a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f^2}\right)\right)\Big/$$

$$\left(b\left(\frac{-a e^{-e}-e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}}{b}-\frac{-a e^{-e}+e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}}{b}\right)\right)\Bigg)+2\,a\,c\,d\,f$$

$$\left(-\left(\left(e^{2e}\left(-a e^{-e}+e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}\right)\left(\frac{x^2}{2\left(a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right)}-\frac{x\,\text{Log}\left[1+\frac{b e^{2e+fx}}{a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f}-\frac{\text{PolyLog}\left[2,-\frac{b e^{2e+fx}}{a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f^2}\right)\right)\Big/\right.\right.$$

$$\left.\left.\left(b\left(\frac{-a e^{-e}-e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}}{b}-\frac{-a e^{-e}+e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}}{b}\right)\right)\right)+\right.$$

$$\left(e^{2e}\left(-a e^{-e}-e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}\right)\left(\frac{x^2}{2\left(a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right)}-\frac{x\,\text{Log}\left[1+\frac{b e^{2e+fx}}{a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f}-\frac{\text{PolyLog}\left[2,-\frac{b e^{2e+fx}}{a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f^2}\right)\right)\Big/$$

$$\left(b\left(\frac{-a e^{-e}-e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}}{b}-\frac{-a e^{-e}+e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}}{b}\right)\right)\Bigg)+$$

$$a\,d^2 f\left(-\left(\left(\left(-a e^{-e}+e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}\right)\left(\frac{x^3}{3\left(a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right)}-\frac{x^2\,\text{Log}\left[1+\frac{b e^{2e+fx}}{a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f}-\frac{2\,x\,\text{PolyLog}\left[2,-\frac{b e^{2e+fx}}{a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f^2}+\right.\right.\right.\right.$$

$$\left.\left.\left.\frac{2\,\text{PolyLog}\left[3,-\frac{b e^{2e+fx}}{a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}-\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f^3}\right)\right)\Big/\left(b\left(\frac{-a e^{-e}-e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}}{b}-\frac{-a e^{-e}+e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}}{b}\right)\right)\right)+$$

$$\left(\left(-a e^{-e}-e^{-2e}\sqrt{a^2 e^{2e}-b^2 e^{2e}}\right)\left(\frac{x^3}{3\left(a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right)}-\frac{x^2\,\text{Log}\left[1+\frac{b e^{2e+fx}}{a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f}-\frac{2\,x\,\text{PolyLog}\left[2,-\frac{b e^{2e+fx}}{a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}}\right]}{\left(a e^{e}+\sqrt{-\left(-a^2+b^2\right) e^{2e}}\right) f^2}+\right.\right.$$

$$\left.\left.\frac{2\,\text{PolyLog}\left[3,-\frac{b\,\mathrm{e}^{2e+fx}}{a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\mathrm{e}^{2e}}}\right]}{\left(a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\mathrm{e}^{2e}}\right)f^3}\right)\right)\Bigg/\left(b\left(\frac{-a\,\mathrm{e}^{-e}-\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}}{b}-\frac{-a\,\mathrm{e}^{-e}+\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}}{b}\right)\right)\right)+a\,d^2\,f$$

$$\left(-\left(\left(\mathrm{e}^{2e}\left(-a\,\mathrm{e}^{-e}+\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}\right)\left(\frac{x^3}{3\left(a\,\mathrm{e}^{e}-\sqrt{-\left(-a^2+b^2\right)\mathrm{e}^{2e}}\right)}-\frac{x^2\,\text{Log}\left[1+\frac{b\,\mathrm{e}^{2e+fx}}{a\,\mathrm{e}^{e}-\sqrt{-\left(-a^2+b^2\right)\mathrm{e}^{2e}}}\right]}{\left(a\,\mathrm{e}^{e}-\sqrt{-\left(-a^2+b^2\right)\mathrm{e}^{2e}}\right)f}-\frac{2\,x\,\text{PolyLog}\left[2,-\frac{b\,\mathrm{e}^{2e+fx}}{a\,\mathrm{e}^{e}-\sqrt{-\left(-a^2+b^2\right)\mathrm{e}^{2e}}}\right]}{\left(a\,\mathrm{e}^{e}-\sqrt{-\left(-a^2+b^2\right)\mathrm{e}^{2e}}\right)f^2}+\right.\right.\right.\right.$$

$$\left.\left.\frac{2\,\text{PolyLog}\left[3,-\frac{b\,\mathrm{e}^{2e+fx}}{a\,\mathrm{e}^{e}-\sqrt{-\left(-a^2+b^2\right)\mathrm{e}^{2e}}}\right]}{\left(a\,\mathrm{e}^{e}-\sqrt{-\left(-a^2+b^2\right)\mathrm{e}^{2e}}\right)f^3}\right)\right)\Bigg/\left(b\left(\frac{-a\,\mathrm{e}^{-e}-\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}}{b}-\frac{-a\,\mathrm{e}^{-e}+\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}}{b}\right)\right)+$$

$$\left(\mathrm{e}^{2e}\left(-a\,\mathrm{e}^{-e}-\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}\right)\left(\frac{x^3}{3\left(a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\mathrm{e}^{2e}}\right)}-\frac{x^2\,\text{Log}\left[1+\frac{b\,\mathrm{e}^{2e+fx}}{a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\mathrm{e}^{2e}}}\right]}{\left(a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\mathrm{e}^{2e}}\right)f}-\frac{2\,x\,\text{PolyLog}\left[2,-\frac{b\,\mathrm{e}^{2e+fx}}{a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\mathrm{e}^{2e}}}\right]}{\left(a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\mathrm{e}^{2e}}\right)f^2}+\right.\right.$$

$$\left.\left.\left.\frac{2\,\text{PolyLog}\left[3,-\frac{b\,\mathrm{e}^{2e+fx}}{a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\mathrm{e}^{2e}}}\right]}{\left(a\,\mathrm{e}^{e}+\sqrt{-\left(-a^2+b^2\right)\mathrm{e}^{2e}}\right)f^3}\right)\right)\Bigg/\left(b\left(\frac{-a\,\mathrm{e}^{-e}-\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}}{b}-\frac{-a\,\mathrm{e}^{-e}+\mathrm{e}^{-2e}\sqrt{a^2\,\mathrm{e}^{2e}-b^2\,\mathrm{e}^{2e}}}{b}\right)\right)\right)+$$

$$\left(\text{Sech}[e]\left(a\,c^2\,\text{Sinh}[e]+2\,a\,c\,d\,x\,\text{Sinh}[e]+a\,d^2\,x^2\,\text{Sinh}[e]-b\,c^2\,\text{Sinh}[f\,x]-2\,b\,c\,d\,x\,\text{Sinh}[f\,x]-b\,d^2\,x^2\,\text{Sinh}[f\,x]\right)\right)\Big/\left((a-b)(a+b)\,f\,(a+b\,\text{Cosh}[e+f\,x])\right)$$

## Problem 180: Result more than twice size of optimal antiderivative.

$$\int (c+d\,x)^m\,(a+b\,\text{Cosh}[e+f\,x])^2\,dx$$

Optimal (type 4, 282 leaves, 10 steps):

$$\frac{a^2 (c+d x)^{1+m}}{d (1+m)}+\frac{b^2 (c+d x)^{1+m}}{2 d (1+m)}+\frac{2^{-3-m} b^2 e^{2 e-\frac{2 c f}{d}} (c+d x)^m \left(-\frac{f (c+d x)}{d}\right)^{-m} \text{Gamma}\left[1+m,-\frac{2 f (c+d x)}{d}\right]}{f}+$$
$$\frac{a b e^{e-\frac{c f}{d}} (c+d x)^m \left(-\frac{f (c+d x)}{d}\right)^{-m} \text{Gamma}\left[1+m,-\frac{f (c+d x)}{d}\right]}{f}-\frac{a b e^{-e+\frac{c f}{d}} (c+d x)^m \left(\frac{f (c+d x)}{d}\right)^{-m} \text{Gamma}\left[1+m,\frac{f (c+d x)}{d}\right]}{f}-$$
$$\frac{2^{-3-m} b^2 e^{-2 e+\frac{2 c f}{d}} (c+d x)^m \left(\frac{f (c+d x)}{d}\right)^{-m} \text{Gamma}\left[1+m,\frac{2 f (c+d x)}{d}\right]}{f}$$

Result (type 4, 650 leaves) :

$$\frac{1}{d f (1+m)} 2^{-3-m} (c+d x)^m \left(-\frac{f^2 (c+d x)^2}{d^2}\right)^{-m}$$
$$\left(2^{3+m} a^2 c f \left(-\frac{f^2 (c+d x)^2}{d^2}\right)^m+2^{2+m} b^2 c f \left(-\frac{f^2 (c+d x)^2}{d^2}\right)^m+2^{3+m} a^2 d f x \left(-\frac{f^2 (c+d x)^2}{d^2}\right)^m+2^{2+m} b^2 d f x \left(-\frac{f^2 (c+d x)^2}{d^2}\right)^m-\right.$$
$$2^{3+m} a b d \left(-\frac{f (c+d x)}{d}\right)^m \text{Cosh}\left[e-\frac{c f}{d}\right] \text{Gamma}\left[1+m,\frac{f (c+d x)}{d}\right]-2^{3+m} a b d m \left(-\frac{f (c+d x)}{d}\right)^m \text{Cosh}\left[e-\frac{c f}{d}\right] \text{Gamma}\left[1+m,\frac{f (c+d x)}{d}\right]-$$
$$b^2 d \left(-\frac{f (c+d x)}{d}\right)^m \text{Cosh}\left[2 e-\frac{2 c f}{d}\right] \text{Gamma}\left[1+m,\frac{2 f (c+d x)}{d}\right]-b^2 d m \left(-\frac{f (c+d x)}{d}\right)^m \text{Cosh}\left[2 e-\frac{2 c f}{d}\right] \text{Gamma}\left[1+m,\frac{2 f (c+d x)}{d}\right]+$$
$$b^2 d \left(-\frac{f (c+d x)}{d}\right)^m \text{Gamma}\left[1+m,\frac{2 f (c+d x)}{d}\right] \text{Sinh}\left[2 e-\frac{2 c f}{d}\right]+b^2 d m \left(-\frac{f (c+d x)}{d}\right)^m \text{Gamma}\left[1+m,\frac{2 f (c+d x)}{d}\right] \text{Sinh}\left[2 e-\frac{2 c f}{d}\right]+$$
$$b^2 d (1+m) \left(f \left(\frac{c}{d}+x\right)\right)^m \text{Gamma}\left[1+m,-\frac{2 f (c+d x)}{d}\right] \left(\text{Cosh}\left[2 e-\frac{2 c f}{d}\right]+\text{Sinh}\left[2 e-\frac{2 c f}{d}\right]\right)+$$
$$2^{3+m} a b d \left(-\frac{f (c+d x)}{d}\right)^m \text{Gamma}\left[1+m,\frac{f (c+d x)}{d}\right] \text{Sinh}\left[e-\frac{c f}{d}\right]+2^{3+m} a b d m \left(-\frac{f (c+d x)}{d}\right)^m \text{Gamma}\left[1+m,\frac{f (c+d x)}{d}\right] \text{Sinh}\left[e-\frac{c f}{d}\right]+$$
$$\left.2^{3+m} a b d (1+m) \left(f \left(\frac{c}{d}+x\right)\right)^m \text{Gamma}\left[1+m,-\frac{f (c+d x)}{d}\right] \left(\text{Cosh}\left[e-\frac{c f}{d}\right]+\text{Sinh}\left[e-\frac{c f}{d}\right]\right)\right)$$

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# Test results for the 111 problems in "6.2.2 (e x)^m (a+b x^n)^p cosh.m"

## Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cosh}[c+d x]}{x (a+b x)^3} \, dx$$

Optimal (type 4, 262 leaves, 17 steps) :

$$\frac{\text{Cosh}[c+d x]}{2 a (a+b x)^2}+\frac{\text{Cosh}[c+d x]}{a^2 (a+b x)}+\frac{\text{Cosh}[c] \text{CoshIntegral}[d x]}{a^3}-\frac{\text{Cosh}\left[c-\frac{a d}{b}\right] \text{CoshIntegral}\left[\frac{a d}{b}+d x\right]}{a^3}-\frac{d^2 \text{Cosh}\left[c-\frac{a d}{b}\right] \text{CoshIntegral}\left[\frac{a d}{b}+d x\right]}{2 a b^2}-\frac{d \text{CoshIntegral}\left[\frac{a d}{b}+d x\right] \text{Sinh}\left[c-\frac{a d}{b}\right]}{a^2 b}+\frac{d \text{Sinh}[c+d x]}{2 a b (a+b x)}+\frac{\text{Sinh}[c] \text{SinhIntegral}[d x]}{a^3}-\frac{d \text{Cosh}\left[c-\frac{a d}{b}\right] \text{SinhIntegral}\left[\frac{a d}{b}+d x\right]}{a^2 b}-\frac{\text{Sinh}\left[c-\frac{a d}{b}\right] \text{SinhIntegral}\left[\frac{a d}{b}+d x\right]}{a^3}-\frac{d^2 \text{Sinh}\left[c-\frac{a d}{b}\right] \text{SinhIntegral}\left[\frac{a d}{b}+d x\right]}{2 a b^2}$$

Result (type 4, 614 leaves):

$$-\frac{1}{2 a^3 b^2 (a+b x)^2}\left(-3 a^2 b^2 \text{Cosh}[c+d x]-2 a b^3 x \text{Cosh}[c+d x]-2 b^2 (a+b x)^2 \text{Cosh}[c] \text{CoshIntegral}[d x]+2 b^2 (a+b x)^2 \text{Cosh}\left[c-\frac{a d}{b}\right] \text{CoshIntegral}\left[d \left(\frac{a}{b}+x\right)\right]+\right.$$
$$a^4 d^2 \text{Cosh}\left[c-\frac{a d}{b}\right] \text{CoshIntegral}\left[\frac{d (a+b x)}{b}\right]+2 a^3 b d^2 x \text{Cosh}\left[c-\frac{a d}{b}\right] \text{CoshIntegral}\left[\frac{d (a+b x)}{b}\right]+$$
$$a^2 b^2 d^2 x^2 \text{Cosh}\left[c-\frac{a d}{b}\right] \text{CoshIntegral}\left[\frac{d (a+b x)}{b}\right]+2 a^3 b d \text{CoshIntegral}\left[\frac{d (a+b x)}{b}\right] \text{Sinh}\left[c-\frac{a d}{b}\right]+$$
$$4 a^2 b^2 d x \text{CoshIntegral}\left[\frac{d (a+b x)}{b}\right] \text{Sinh}\left[c-\frac{a d}{b}\right]+2 a b^3 d x^2 \text{CoshIntegral}\left[\frac{d (a+b x)}{b}\right] \text{Sinh}\left[c-\frac{a d}{b}\right]-$$
$$a^3 b d \text{Sinh}[c+d x]-a^2 b^2 d x \text{Sinh}[c+d x]-2 a^2 b^2 \text{Sinh}[c] \text{SinhIntegral}[d x]-4 a b^3 x \text{Sinh}[c] \text{SinhIntegral}[d x]-$$
$$2 b^4 x^2 \text{Sinh}[c] \text{SinhIntegral}[d x]+2 a^2 b^2 \text{Sinh}\left[c-\frac{a d}{b}\right] \text{SinhIntegral}\left[d \left(\frac{a}{b}+x\right)\right]+$$
$$4 a b^3 x \text{Sinh}\left[c-\frac{a d}{b}\right] \text{SinhIntegral}\left[d \left(\frac{a}{b}+x\right)\right]+2 b^4 x^2 \text{Sinh}\left[c-\frac{a d}{b}\right] \text{SinhIntegral}\left[d \left(\frac{a}{b}+x\right)\right]+$$
$$2 a^3 b d \text{Cosh}\left[c-\frac{a d}{b}\right] \text{SinhIntegral}\left[\frac{d (a+b x)}{b}\right]+4 a^2 b^2 d x \text{Cosh}\left[c-\frac{a d}{b}\right] \text{SinhIntegral}\left[\frac{d (a+b x)}{b}\right]+$$
$$2 a b^3 d x^2 \text{Cosh}\left[c-\frac{a d}{b}\right] \text{SinhIntegral}\left[\frac{d (a+b x)}{b}\right]+a^4 d^2 \text{Sinh}\left[c-\frac{a d}{b}\right] \text{SinhIntegral}\left[\frac{d (a+b x)}{b}\right]+$$
$$\left.2 a^3 b d^2 x \text{Sinh}\left[c-\frac{a d}{b}\right] \text{SinhIntegral}\left[\frac{d (a+b x)}{b}\right]+a^2 b^2 d^2 x^2 \text{Sinh}\left[c-\frac{a d}{b}\right] \text{SinhIntegral}\left[\frac{d (a+b x)}{b}\right]\right)$$

## Problem 38: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cosh}[c+d x]}{x^2 (a+b x)^3} \, dx$$

Optimal (type 4, 298 leaves, 21 steps):

$$-\frac{\text{Cosh}[c+d x]}{a^3 x}-\frac{b\,\text{Cosh}[c+d x]}{2 a^2 (a+b x)^2}-\frac{2 b\,\text{Cosh}[c+d x]}{a^3 (a+b x)}-\frac{3 b\,\text{Cosh}[c]\,\text{CoshIntegral}[d x]}{a^4}+$$
$$\frac{3 b\,\text{Cosh}\left[c-\frac{a d}{b}\right]\text{CoshIntegral}\left[\frac{a d}{b}+d x\right]}{a^4}+\frac{d^2\,\text{Cosh}\left[c-\frac{a d}{b}\right]\text{CoshIntegral}\left[\frac{a d}{b}+d x\right]}{2 a^2 b}+\frac{d\,\text{CoshIntegral}[d x]\,\text{Sinh}[c]}{a^3}+$$
$$\frac{2 d\,\text{CoshIntegral}\left[\frac{a d}{b}+d x\right]\text{Sinh}\left[c-\frac{a d}{b}\right]}{a^3}-\frac{d\,\text{Sinh}[c+d x]}{2 a^2 (a+b x)}+\frac{d\,\text{Cosh}[c]\,\text{SinhIntegral}[d x]}{a^3}-\frac{3 b\,\text{Sinh}[c]\,\text{SinhIntegral}[d x]}{a^4}+$$
$$\frac{2 d\,\text{Cosh}\left[c-\frac{a d}{b}\right]\text{SinhIntegral}\left[\frac{a d}{b}+d x\right]}{a^3}+\frac{3 b\,\text{Sinh}\left[c-\frac{a d}{b}\right]\text{SinhIntegral}\left[\frac{a d}{b}+d x\right]}{a^4}+\frac{d^2\,\text{Sinh}\left[c-\frac{a d}{b}\right]\text{SinhIntegral}\left[\frac{a d}{b}+d x\right]}{2 a^2 b}$$

Result (type 4, 710 leaves):

$$\frac{1}{2 a^4 b x (a+b x)^2}\Bigg(-2 a^3 b\,\text{Cosh}[c+d x]-9 a^2 b^2 x\,\text{Cosh}[c+d x]-6 a b^3 x^2\,\text{Cosh}[c+d x]+6 b^2 x (a+b x)^2\,\text{Cosh}\left[c-\frac{a d}{b}\right]\text{CoshIntegral}\left[d\left(\frac{a}{b}+x\right)\right]+$$
$$a^4 d^2 x\,\text{Cosh}\left[c-\frac{a d}{b}\right]\text{CoshIntegral}\left[\frac{d (a+b x)}{b}\right]+2 a^3 b d^2 x^2\,\text{Cosh}\left[c-\frac{a d}{b}\right]\text{CoshIntegral}\left[\frac{d (a+b x)}{b}\right]+$$
$$a^2 b^2 d^2 x^3\,\text{Cosh}\left[c-\frac{a d}{b}\right]\text{CoshIntegral}\left[\frac{d (a+b x)}{b}\right]+2 b x (a+b x)^2\,\text{CoshIntegral}[d x]\,(-3 b\,\text{Cosh}[c]+a d\,\text{Sinh}[c])+$$
$$4 a^3 b d x\,\text{CoshIntegral}\left[\frac{d (a+b x)}{b}\right]\text{Sinh}\left[c-\frac{a d}{b}\right]+8 a^2 b^2 d x^2\,\text{CoshIntegral}\left[\frac{d (a+b x)}{b}\right]\text{Sinh}\left[c-\frac{a d}{b}\right]+$$
$$4 a b^3 d x^3\,\text{CoshIntegral}\left[\frac{d (a+b x)}{b}\right]\text{Sinh}\left[c-\frac{a d}{b}\right]-a^3 b d x\,\text{Sinh}[c+d x]-a^2 b^2 d x^2\,\text{Sinh}[c+d x]+2 a^3 b d x\,\text{Cosh}[c]\,\text{SinhIntegral}[d x]+$$
$$4 a^2 b^2 d x^2\,\text{Cosh}[c]\,\text{SinhIntegral}[d x]+2 a b^3 d x^3\,\text{Cosh}[c]\,\text{SinhIntegral}[d x]-6 a^2 b^2 x\,\text{Sinh}[c]\,\text{SinhIntegral}[d x]-$$
$$12 a b^3 x^2\,\text{Sinh}[c]\,\text{SinhIntegral}[d x]-6 b^4 x^3\,\text{Sinh}[c]\,\text{SinhIntegral}[d x]+6 a^2 b^2 x\,\text{Sinh}\left[c-\frac{a d}{b}\right]\text{SinhIntegral}\left[d\left(\frac{a}{b}+x\right)\right]+$$
$$12 a b^3 x^2\,\text{Sinh}\left[c-\frac{a d}{b}\right]\text{SinhIntegral}\left[d\left(\frac{a}{b}+x\right)\right]+6 b^4 x^3\,\text{Sinh}\left[c-\frac{a d}{b}\right]\text{SinhIntegral}\left[d\left(\frac{a}{b}+x\right)\right]+$$
$$4 a^3 b d x\,\text{Cosh}\left[c-\frac{a d}{b}\right]\text{SinhIntegral}\left[\frac{d (a+b x)}{b}\right]+8 a^2 b^2 d x^2\,\text{Cosh}\left[c-\frac{a d}{b}\right]\text{SinhIntegral}\left[\frac{d (a+b x)}{b}\right]+$$
$$4 a b^3 d x^3\,\text{Cosh}\left[c-\frac{a d}{b}\right]\text{SinhIntegral}\left[\frac{d (a+b x)}{b}\right]+a^4 d^2 x\,\text{Sinh}\left[c-\frac{a d}{b}\right]\text{SinhIntegral}\left[\frac{d (a+b x)}{b}\right]+$$
$$2 a^3 b d^2 x^2\,\text{Sinh}\left[c-\frac{a d}{b}\right]\text{SinhIntegral}\left[\frac{d (a+b x)}{b}\right]+a^2 b^2 d^2 x^3\,\text{Sinh}\left[c-\frac{a d}{b}\right]\text{SinhIntegral}\left[\frac{d (a+b x)}{b}\right]\Bigg)$$

## Problem 57: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^4\,\text{Cosh}[c+d x]}{a+b x^2}\,dx$$

Optimal (type 4, 273 leaves, 14 steps):

$$-\frac{2\,x\,\text{Cosh}[c+d\,x]}{b\,d^2}+\frac{(-a)^{3/2}\,\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{2\,b^{5/2}}-\frac{(-a)^{3/2}\,\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{2\,b^{5/2}}+\frac{2\,\text{Sinh}[c+d\,x]}{b\,d^3}-\frac{a\,\text{Sinh}[c+d\,x]}{b^2\,d}+\frac{x^2\,\text{Sinh}[c+d\,x]}{b\,d}-\frac{(-a)^{3/2}\,\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{2\,b^{5/2}}-\frac{(-a)^{3/2}\,\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{2\,b^{5/2}}$$

Result (type 4, 274 leaves):

$$\frac{1}{2\,b^{5/2}\,d^3}\left(-4\,b^{3/2}\,d\,x\,\text{Cosh}[c+d\,x]+i\,a^{3/2}\,d^3\,\text{Cosh}\left[c-\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]-i\,a^{3/2}\,d^3\,\text{Cosh}\left[c+\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]+4\,b^{3/2}\,\text{Sinh}[c+d\,x]-2\,a\,\sqrt{b}\,d^2\,\text{Sinh}[c+d\,x]+2\,b^{3/2}\,d^2\,x^2\,\text{Sinh}[c+d\,x]-a^{3/2}\,d^3\,\text{Sinh}\left[c-\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i\,d\,x\right]-a^{3/2}\,d^3\,\text{Sinh}\left[c+\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\right)$$

## Problem 58: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^3\,\text{Cosh}[c+d\,x]}{a+b\,x^2}\,dx$$

Optimal (type 4, 209 leaves, 12 steps):

$$-\frac{\text{Cosh}[c+d\,x]}{b\,d^2}-\frac{a\,\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{2\,b^2}-\frac{a\,\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{2\,b^2}+\frac{x\,\text{Sinh}[c+d\,x]}{b\,d}+\frac{a\,\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{2\,b^2}-\frac{a\,\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{2\,b^2}$$

Result (type 4, 210 leaves):

$$-\frac{1}{2\,b^2\,d^2}\left(2\,b\,\text{Cosh}[c+d\,x]+a\,d^2\,\text{Cosh}\left[c-\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]+a\,d^2\,\text{Cosh}\left[c+\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]-2\,b\,d\,x\,\text{Sinh}[c+d\,x]+i\,a\,d^2\,\text{Sinh}\left[c-\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i\,d\,x\right]-i\,a\,d^2\,\text{Sinh}\left[c+\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\right)$$

## Problem 59: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 \operatorname{Cosh}[c+d x]}{a+b x^2} \, dx$$

Optimal (type 4, 226 leaves, 11 steps):

$$\frac{\sqrt{-a}\, \operatorname{Cosh}\left[c+\frac{\sqrt{-a}\, d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}-d x\right]}{2 b^{3/2}}-\frac{\sqrt{-a}\, \operatorname{Cosh}\left[c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}+d x\right]}{2 b^{3/2}}+\frac{\operatorname{Sinh}[c+d x]}{b d}-\frac{\sqrt{-a}\, \operatorname{Sinh}\left[c+\frac{\sqrt{-a}\, d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}-d x\right]}{2 b^{3/2}}-\frac{\sqrt{-a}\, \operatorname{Sinh}\left[c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}+d x\right]}{2 b^{3/2}}$$

Result (type 4, 213 leaves):

$$\frac{1}{2 b^{3/2} d}\left(-i \sqrt{a}\, d \operatorname{Cosh}\left[c-\frac{i \sqrt{a}\, d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[-\frac{\sqrt{a}\, d}{\sqrt{b}}+i d x\right]+i \sqrt{a}\, d \operatorname{Cosh}\left[c+\frac{i \sqrt{a}\, d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}+i d x\right]+2 \sqrt{b}\, \operatorname{Sinh}[c+d x]+\sqrt{a}\, d \operatorname{Sinh}\left[c-\frac{i \sqrt{a}\, d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}-i d x\right]+\sqrt{a}\, d \operatorname{Sinh}\left[c+\frac{i \sqrt{a}\, d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}+i d x\right]\right)$$

## Problem 60: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x \operatorname{Cosh}[c+d x]}{a+b x^2} \, dx$$

Optimal (type 4, 177 leaves, 8 steps):

$$\frac{\operatorname{Cosh}\left[c+\frac{\sqrt{-a}\, d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}-d x\right]}{2 b}+\frac{\operatorname{Cosh}\left[c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}+d x\right]}{2 b}-\frac{\operatorname{Sinh}\left[c+\frac{\sqrt{-a}\, d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}-d x\right]}{2 b}+\frac{\operatorname{Sinh}\left[c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}+d x\right]}{2 b}$$

Result (type 4, 171 leaves):

$$\frac{1}{2 b}\left(\operatorname{Cosh}\left[c-\frac{i \sqrt{a}\, d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[-\frac{\sqrt{a}\, d}{\sqrt{b}}+i d x\right]+\operatorname{Cosh}\left[c+\frac{i \sqrt{a}\, d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}+i d x\right]+i \left(\operatorname{Sinh}\left[c-\frac{i \sqrt{a}\, d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}-i d x\right]-\operatorname{Sinh}\left[c+\frac{i \sqrt{a}\, d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}+i d x\right]\right)\right)$$

## Problem 61: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Cosh}[c+d\,x]}{a+b\,x^2}\,dx$$

Optimal (type 4, 213 leaves, 8 steps):

$$\frac{\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{2\sqrt{-a}\,\sqrt{b}}-\frac{\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{2\sqrt{-a}\,\sqrt{b}}-\frac{\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{2\sqrt{-a}\,\sqrt{b}}-\frac{\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{2\sqrt{-a}\,\sqrt{b}}$$

Result (type 4, 180 leaves):

$$\frac{1}{2\sqrt{a}\,\sqrt{b}}\,i\left(\text{Cosh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]-\text{Cosh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]+i\left(\text{Sinh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i\,d\,x\right]+\text{Sinh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\right)\right)$$

## Problem 62: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Cosh}[c+d\,x]}{x\left(a+b\,x^2\right)}\,dx$$

Optimal (type 4, 197 leaves, 13 steps):

$$\frac{\text{Cosh}[c]\,\text{CoshIntegral}[d\,x]}{a}-\frac{\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{2\,a}-\frac{\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{2\,a}+\frac{\text{Sinh}[c]\,\text{SinhIntegral}[d\,x]}{a}+\frac{\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{2\,a}-\frac{\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{2\,a}$$

Result (type 4, 187 leaves):

$$-\frac{1}{2\,a}\left(-2\,\text{Cosh}[c]\,\text{CoshIntegral}[d\,x]+\text{Cosh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]+\text{Cosh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]-2\,\text{Sinh}[c]\,\text{SinhIntegral}[d\,x]+i\,\text{Sinh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i\,d\,x\right]-i\,\text{Sinh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\right)$$

## Problem 63: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Cosh}[c+d x]}{x^2 \left(a+b x^2\right)} \, dx$$

Optimal (type 4, 249 leaves, 14 steps):

$$-\frac{\text{Cosh}[c+d x]}{a x}+\frac{\sqrt{b} \, \text{Cosh}\left[c+\frac{\sqrt{-a} \, d}{\sqrt{b}}\right] \text{CoshIntegral}\left[\frac{\sqrt{-a} \, d}{\sqrt{b}}-d x\right]}{2 (-a)^{3/2}}-\frac{\sqrt{b} \, \text{Cosh}\left[c-\frac{\sqrt{-a} \, d}{\sqrt{b}}\right] \text{CoshIntegral}\left[\frac{\sqrt{-a} \, d}{\sqrt{b}}+d x\right]}{2 (-a)^{3/2}}+\frac{d \, \text{CoshIntegral}[d x] \, \text{Sinh}[c]}{a}+\frac{d \, \text{Cosh}[c] \, \text{SinhIntegral}[d x]}{a}-\frac{\sqrt{b} \, \text{Sinh}\left[c+\frac{\sqrt{-a} \, d}{\sqrt{b}}\right] \text{SinhIntegral}\left[\frac{\sqrt{-a} \, d}{\sqrt{b}}-d x\right]}{2 (-a)^{3/2}}-\frac{\sqrt{b} \, \text{Sinh}\left[c-\frac{\sqrt{-a} \, d}{\sqrt{b}}\right] \text{SinhIntegral}\left[\frac{\sqrt{-a} \, d}{\sqrt{b}}+d x\right]}{2 (-a)^{3/2}}$$

Result (type 4, 243 leaves):

$$\frac{1}{2 a^{3/2} x}\left(-2 \sqrt{a} \, \text{Cosh}[c+d x]-i \sqrt{b} \, x \, \text{Cosh}\left[c-\frac{i \sqrt{a} \, d}{\sqrt{b}}\right] \text{CosIntegral}\left[-\frac{\sqrt{a} \, d}{\sqrt{b}}+i d x\right]+i \sqrt{b} \, x \, \text{Cosh}\left[c+\frac{i \sqrt{a} \, d}{\sqrt{b}}\right] \text{CosIntegral}\left[\frac{\sqrt{a} \, d}{\sqrt{b}}+i d x\right]+2 \sqrt{a} \, d x \, \text{CoshIntegral}[d x] \, \text{Sinh}[c]+2 \sqrt{a} \, d x \, \text{Cosh}[c] \, \text{SinhIntegral}[d x]+\sqrt{b} \, x \, \text{Sinh}\left[c-\frac{i \sqrt{a} \, d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a} \, d}{\sqrt{b}}-i d x\right]+\sqrt{b} \, x \, \text{Sinh}\left[c+\frac{i \sqrt{a} \, d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a} \, d}{\sqrt{b}}+i d x\right]\right)$$

## Problem 64: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Cosh}[c+d x]}{x^3 \left(a+b x^2\right)} \, dx$$

Optimal (type 4, 270 leaves, 18 steps):

$$-\frac{\text{Cosh}[c+d x]}{2 a x^2}-\frac{b \, \text{Cosh}[c] \, \text{CoshIntegral}[d x]}{a^2}+\frac{d^2 \, \text{Cosh}[c] \, \text{CoshIntegral}[d x]}{2 a}+\frac{b \, \text{Cosh}\left[c+\frac{\sqrt{-a} \, d}{\sqrt{b}}\right] \text{CoshIntegral}\left[\frac{\sqrt{-a} \, d}{\sqrt{b}}-d x\right]}{2 a^2}+\frac{b \, \text{Cosh}\left[c-\frac{\sqrt{-a} \, d}{\sqrt{b}}\right] \text{CoshIntegral}\left[\frac{\sqrt{-a} \, d}{\sqrt{b}}+d x\right]}{2 a^2}-\frac{d \, \text{Sinh}[c+d x]}{2 a x}-\frac{b \, \text{Sinh}[c] \, \text{SinhIntegral}[d x]}{a^2}+\frac{d^2 \, \text{Sinh}[c] \, \text{SinhIntegral}[d x]}{2 a}-\frac{b \, \text{Sinh}\left[c+\frac{\sqrt{-a} \, d}{\sqrt{b}}\right] \text{SinhIntegral}\left[\frac{\sqrt{-a} \, d}{\sqrt{b}}-d x\right]}{2 a^2}+\frac{b \, \text{Sinh}\left[c-\frac{\sqrt{-a} \, d}{\sqrt{b}}\right] \text{SinhIntegral}\left[\frac{\sqrt{-a} \, d}{\sqrt{b}}+d x\right]}{2 a^2}$$

Result (type 4, 257 leaves):

$$\frac{1}{2 a^2 x^2}\left(-a \operatorname{Cosh}[c+d x]-\left(2 b-a d^2\right) x^2 \operatorname{Cosh}[c] \operatorname{CoshIntegral}[d x]+b x^2 \operatorname{Cosh}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[-\frac{\sqrt{a} d}{\sqrt{b}}+i d x\right]+\right.$$
$$b x^2 \operatorname{Cosh}\left[c+\frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}}+i d x\right]-a d x \operatorname{Sinh}[c+d x]-2 b x^2 \operatorname{Sinh}[c] \operatorname{SinhIntegral}[d x]+a d^2 x^2 \operatorname{Sinh}[c] \operatorname{SinhIntegral}[d x]+$$
$$\left.i b x^2 \operatorname{Sinh}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}}-i d x\right]-i b x^2 \operatorname{Sinh}\left[c+\frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}}+i d x\right]\right)$$

## Problem 65: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 \operatorname{Cosh}[c+d x]}{\left(a+b x^2\right)^2} \, dx$$

Optimal (type 4, 449 leaves, 24 steps):

$$\frac{x \operatorname{Cosh}[c+d x]}{2 b^2}-\frac{x^3 \operatorname{Cosh}[c+d x]}{2 b \left(a+b x^2\right)}+\frac{3 \sqrt{-a} \operatorname{Cosh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{4 b^{5/2}}-\frac{3 \sqrt{-a} \operatorname{Cosh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{4 b^{5/2}}-$$
$$\frac{a d \operatorname{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right] \operatorname{Sinh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right]}{4 b^3}-\frac{a d \operatorname{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right] \operatorname{Sinh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right]}{4 b^3}+\frac{\operatorname{Sinh}[c+d x]}{b^2 d}+$$
$$\frac{a d \operatorname{Cosh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{4 b^3}-\frac{3 \sqrt{-a} \operatorname{Sinh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{4 b^{5/2}}-$$
$$\frac{a d \operatorname{Cosh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{4 b^3}-\frac{3 \sqrt{-a} \operatorname{Sinh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{4 b^{5/2}}$$

Result (type 4, 621 leaves):

$$\frac{1}{4\,b^2}\left(2\,\text{Cosh}[d\,x]\left(\frac{a\,x\,\text{Cosh}[c]}{a+b\,x^2}+\frac{2\,\text{Sinh}[c]}{d}\right)+\right.$$
$$2\left(\frac{2\,\text{Cosh}[c]}{d}+\frac{a\,x\,\text{Sinh}[c]}{a+b\,x^2}\right)\text{Sinh}[d\,x]-\frac{1}{\sqrt{b}}3\,\mathrm{i}\,\sqrt{a}\,\text{Cosh}[c]\left(\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]-\right.$$
$$\left.\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]+\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\left(\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-\mathrm{i}\,d\,x\right]-\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]\right)\right)+$$
$$\frac{1}{b}\mathrm{i}\,a\,d\,\text{Cosh}[c]\left(\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]-\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]+\right.$$
$$\left.\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\left(-\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-\mathrm{i}\,d\,x\right]+\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]\right)\right)-$$
$$\frac{1}{\sqrt{b}}3\,\sqrt{a}\,\text{Sinh}[c]\left(\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]+\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]-\right.$$
$$\left.\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\left(\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-\mathrm{i}\,d\,x\right]+\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]\right)\right)-$$
$$\frac{1}{b}a\,d\,\text{Sinh}[c]\left(\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]+\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]+\right.$$
$$\left.\left.\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\left(\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-\mathrm{i}\,d\,x\right]+\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]\right)\right)\right)$$

## Problem 66: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^3\,\text{Cosh}[c+d\,x]}{\left(a+b\,x^2\right)^2}\,dx$$

Optimal (type 4, 431 leaves, 20 steps):

$$\frac{\text{Cosh}[c+d x]}{2 b^2}-\frac{x^2 \text{Cosh}[c+d x]}{2 b \left(a+b x^2\right)}+\frac{\text{Cosh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{2 b^2}+\frac{\text{Cosh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{2 b^2}-$$
$$\frac{\sqrt{-a}\, d\, \text{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right] \text{Sinh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right]}{4 b^{5/2}}+\frac{\sqrt{-a}\, d\, \text{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right] \text{Sinh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right]}{4 b^{5/2}}-$$
$$\frac{\sqrt{-a}\, d\, \text{Cosh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{4 b^{5/2}}-\frac{\text{Sinh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{2 b^2}-$$
$$\frac{\sqrt{-a}\, d\, \text{Cosh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{4 b^{5/2}}+\frac{\text{Sinh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{2 b^2}$$

Result (type 4, 582 leaves):

$$\frac{1}{4 b^{5/2} \left(a+b x^2\right)}\left(2 a \sqrt{b}\, \text{Cosh}[c+d x]+\left(a+b x^2\right) \text{CosIntegral}\left[-\frac{\sqrt{a} d}{\sqrt{b}}+i d x\right]\left(2 \sqrt{b}\, \text{Cosh}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right]-i \sqrt{a}\, d\, \text{Sinh}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right]\right)+\right.$$
$$\left(a+b x^2\right) \text{CosIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}}+i d x\right]\left(2 \sqrt{b}\, \text{Cosh}\left[c+\frac{i \sqrt{a} d}{\sqrt{b}}\right]+i \sqrt{a}\, d\, \text{Sinh}\left[c+\frac{i \sqrt{a} d}{\sqrt{b}}\right]\right)+$$
$$a^{3/2} d\, \text{Cosh}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}}-i d x\right]+\sqrt{a}\, b\, d\, x^2 \text{Cosh}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}}-i d x\right]+$$
$$2 i a \sqrt{b}\, \text{Sinh}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}}-i d x\right]+2 i b^{3/2} x^2 \text{Sinh}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}}-i d x\right]+$$
$$a^{3/2} d\, \text{Cosh}\left[c+\frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}}+i d x\right]+\sqrt{a}\, b\, d\, x^2 \text{Cosh}\left[c+\frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}}+i d x\right]-$$
$$\left.2 i a \sqrt{b}\, \text{Sinh}\left[c+\frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}}+i d x\right]-2 i b^{3/2} x^2 \text{Sinh}\left[c+\frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}}+i d x\right]\right)$$

## Problem 67: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 \text{Cosh}[c+d x]}{\left(a+b x^2\right)^2}\, dx$$

Optimal (type 4, 416 leaves, 17 steps):

$$-\frac{x\,\mathrm{Cosh}[c+d\,x]}{2\,b\,(a+b\,x^2)}+\frac{\mathrm{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\mathrm{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{4\,\sqrt{-a}\,b^{3/2}}-\frac{\mathrm{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\mathrm{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{4\,\sqrt{-a}\,b^{3/2}}+$$
$$\frac{d\,\mathrm{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]\mathrm{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]}{4\,b^2}+\frac{d\,\mathrm{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]\mathrm{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]}{4\,b^2}-\frac{d\,\mathrm{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\mathrm{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{4\,b^2}-$$
$$\frac{\mathrm{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\mathrm{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{4\,\sqrt{-a}\,b^{3/2}}+\frac{d\,\mathrm{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\mathrm{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{4\,b^2}-\frac{\mathrm{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\mathrm{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{4\,\sqrt{-a}\,b^{3/2}}$$

Result (type 4, 364 leaves):

$$\frac{1}{4\,\sqrt{a}\,b^2\,(a+b\,x^2)}\left(-2\,\sqrt{a}\,b\,x\,\mathrm{Cosh}[c+d\,x]+(a+b\,x^2)\,\mathrm{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\left(i\,\sqrt{b}\,\mathrm{Cosh}\left[c-\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]+\sqrt{a}\,d\,\mathrm{Sinh}\left[c-\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]\right)+\right.$$
$$(a+b\,x^2)\,\mathrm{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\left(-i\,\sqrt{b}\,\mathrm{Cosh}\left[c+\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]+\sqrt{a}\,d\,\mathrm{Sinh}\left[c+\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]\right)+$$
$$(a+b\,x^2)\left(i\,\sqrt{a}\,d\,\mathrm{Cosh}\left[c-\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]-\sqrt{b}\,\mathrm{Sinh}\left[c-\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]\right)\mathrm{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i\,d\,x\right]-$$
$$\left.(a+b\,x^2)\left(i\,\sqrt{a}\,d\,\mathrm{Cosh}\left[c+\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]+\sqrt{b}\,\mathrm{Sinh}\left[c+\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]\right)\mathrm{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\right)$$

## Problem 68: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\,\mathrm{Cosh}[c+d\,x]}{(a+b\,x^2)^2}\,dx$$

Optimal (type 4, 239 leaves, 9 steps):

$$-\frac{\mathrm{Cosh}[c+d\,x]}{2\,b\,(a+b\,x^2)}-\frac{d\,\mathrm{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]\mathrm{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]}{4\,\sqrt{-a}\,b^{3/2}}+\frac{d\,\mathrm{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]\mathrm{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]}{4\,\sqrt{-a}\,b^{3/2}}-$$
$$\frac{d\,\mathrm{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\mathrm{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{4\,\sqrt{-a}\,b^{3/2}}-\frac{d\,\mathrm{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\mathrm{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{4\,\sqrt{-a}\,b^{3/2}}$$

Result (type 4, 239 leaves):

$$\frac{1}{4\sqrt{a}\, b^{3/2}\left(a+b x^2\right)} i \left(d\left(a+b x^2\right)\text{CosIntegral}\left[-\frac{\sqrt{a}\, d}{\sqrt{b}}+i d x\right]\text{Sinh}\left[c-\frac{i\sqrt{a}\, d}{\sqrt{b}}\right]-d\left(a+b x^2\right)\text{CosIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}+i d x\right]\text{Sinh}\left[c+\frac{i\sqrt{a}\, d}{\sqrt{b}}\right]+i\left(2\sqrt{a}\sqrt{b}\,\text{Cosh}[c+d x]+d\left(a+b x^2\right)\text{Cosh}\left[c-\frac{i\sqrt{a}\, d}{\sqrt{b}}\right]\text{SinIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}-i d x\right]+d\left(a+b x^2\right)\text{Cosh}\left[c+\frac{i\sqrt{a}\, d}{\sqrt{b}}\right]\text{SinIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}+i d x\right]\right)\right)$$

## Problem 69: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Cosh}[c+d x]}{\left(a+b x^2\right)^2}\, dx$$

Optimal (type 4, 476 leaves, 18 steps):

$$-\frac{\text{Cosh}[c+d x]}{4 a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\, x\right)}+\frac{\text{Cosh}[c+d x]}{4 a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\, x\right)}-\frac{\text{Cosh}\left[c+\frac{\sqrt{-a}\, d}{\sqrt{b}}\right]\text{CoshIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}-d x\right]}{4(-a)^{3/2}\sqrt{b}}+\frac{\text{Cosh}\left[c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right]\text{CoshIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}+d x\right]}{4(-a)^{3/2}\sqrt{b}}-$$
$$\frac{d\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}+d x\right]\text{Sinh}\left[c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right]}{4 a b}-\frac{d\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}-d x\right]\text{Sinh}\left[c+\frac{\sqrt{-a}\, d}{\sqrt{b}}\right]}{4 a b}+\frac{d\,\text{Cosh}\left[c+\frac{\sqrt{-a}\, d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}-d x\right]}{4 a b}+$$
$$\frac{\text{Sinh}\left[c+\frac{\sqrt{-a}\, d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}-d x\right]}{4(-a)^{3/2}\sqrt{b}}-\frac{d\,\text{Cosh}\left[c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}+d x\right]}{4 a b}+\frac{\text{Sinh}\left[c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}+d x\right]}{4(-a)^{3/2}\sqrt{b}}$$

Result (type 4, 590 leaves):

$$\frac{1}{4\,a^{3/2}\,b\,\left(a+b\,x^2\right)}\left(2\,\sqrt{a}\,b\,x\,\text{Cosh}[c+d\,x]-\left(a+b\,x^2\right)\,\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]\,\left(-\mathrm{i}\,\sqrt{b}\,\text{Cosh}\left[c-\frac{\mathrm{i}\,\sqrt{a}\,d}{\sqrt{b}}\right]+\sqrt{a}\,d\,\text{Sinh}\left[c-\frac{\mathrm{i}\,\sqrt{a}\,d}{\sqrt{b}}\right]\right)-\right.$$
$$\left(a+b\,x^2\right)\,\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]\,\left(\mathrm{i}\,\sqrt{b}\,\text{Cosh}\left[c+\frac{\mathrm{i}\,\sqrt{a}\,d}{\sqrt{b}}\right]+\sqrt{a}\,d\,\text{Sinh}\left[c+\frac{\mathrm{i}\,\sqrt{a}\,d}{\sqrt{b}}\right]\right)-$$
$$\mathrm{i}\,a^{3/2}\,d\,\text{Cosh}\left[c-\frac{\mathrm{i}\,\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-\mathrm{i}\,d\,x\right]-\mathrm{i}\,\sqrt{a}\,b\,d\,x^2\,\text{Cosh}\left[c-\frac{\mathrm{i}\,\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-\mathrm{i}\,d\,x\right]-$$
$$a\,\sqrt{b}\,\text{Sinh}\left[c-\frac{\mathrm{i}\,\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-\mathrm{i}\,d\,x\right]-b^{3/2}\,x^2\,\text{Sinh}\left[c-\frac{\mathrm{i}\,\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-\mathrm{i}\,d\,x\right]+$$
$$\mathrm{i}\,a^{3/2}\,d\,\text{Cosh}\left[c+\frac{\mathrm{i}\,\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]+\mathrm{i}\,\sqrt{a}\,b\,d\,x^2\,\text{Cosh}\left[c+\frac{\mathrm{i}\,\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]-$$
$$\left.a\,\sqrt{b}\,\text{Sinh}\left[c+\frac{\mathrm{i}\,\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]-b^{3/2}\,x^2\,\text{Sinh}\left[c+\frac{\mathrm{i}\,\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]\right)$$

## Problem 70: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\text{Cosh}[c+d\,x]}{x\,\left(a+b\,x^2\right)^2}\,dx$$

Optimal (type 4, 435 leaves, 22 steps):

$$\frac{\text{Cosh}[c+d\,x]}{2\,a\,\left(a+b\,x^2\right)}+\frac{\text{Cosh}[c]\,\text{CoshIntegral}[d\,x]}{a^2}-\frac{\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{2\,a^2}-$$
$$\frac{\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{2\,a^2}-\frac{d\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]\,\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]}{4\,(-a)^{3/2}\,\sqrt{b}}+$$
$$\frac{d\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]\,\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]}{4\,(-a)^{3/2}\,\sqrt{b}}+\frac{\text{Sinh}[c]\,\text{SinhIntegral}[d\,x]}{a^2}-\frac{d\,\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{4\,(-a)^{3/2}\,\sqrt{b}}+$$
$$\frac{\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{2\,a^2}-\frac{d\,\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{4\,(-a)^{3/2}\,\sqrt{b}}-\frac{\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{2\,a^2}$$

Result (type 4, 2464 leaves):

$$\text{Sinh}[c]\left(\frac{\text{SinhIntegral}[d\,x]}{a^2}-\frac{-i\,\text{CoshIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]+\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]}{2\,a^2}-\right.$$

$$\frac{i\sqrt{b}\left(-\frac{\text{Sinh}[d\,x]}{i\sqrt{a}\,\sqrt{b}+b\,x}+\frac{d\left(\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]-i\,\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]\right)}{b}\right)}{4\,a^{3/2}}+$$

$$\frac{-i\,\text{CoshIntegral}\left[-\frac{i\sqrt{a}\,d}{\sqrt{b}}+d\,x\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]+\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}-d\,x\right]}{2\,a^2}+$$

$$\left.\frac{i\sqrt{b}\left(-\frac{\text{Sinh}[d\,x]}{-i\sqrt{a}\,\sqrt{b}+b\,x}+\frac{d\left(\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]-i\,\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}-d\,x\right]\right)}{b}\right)}{4\,a^{3/2}}\right)+$$

$$\text{Cosh}[c]\left(\frac{\text{CoshIntegral}[d\,x]}{a^2}-\frac{i\sqrt{b}\left(-\frac{\text{Cosh}[d\,x]}{i\sqrt{a}\,\sqrt{b}+b\,x}+\frac{d\left(-i\,\text{CoshIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]+\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]\right)}{b}\right)}{4\,a^{3/2}}-\right.$$

$$\frac{\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[-\frac{i\sqrt{a}\,d}{\sqrt{b}}+d\,x\right]-i\,\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}-d\,x\right]}{2\,a^2}+$$

$$\frac{i\sqrt{b}\left(-\frac{\text{Cosh}[d\,x]}{-i\sqrt{a}\,\sqrt{b}+b\,x}-\frac{d\left(-i\,\text{CoshIntegral}\left[d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]+\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}-d\,x\right]\right)}{b}\right)}{4\,a^{3/2}}-$$

$$\left.\frac{\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}+d\,x\right]-i\,\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}+d\,x\right]}{2\,a^2}\right)+$$

$$\frac{1}{2}\left(-\text{Cosh}[c]\left(\frac{\text{SinhIntegral}[d\,x]}{a^2}-\frac{-i\,\text{CoshIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]+\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]}{2\,a^2}-\right.\right.$$

$$\frac{i\sqrt{b}\left(-\frac{\text{Sinh}[d\,x]}{i\sqrt{a}\sqrt{b}+b\,x}+\frac{d\left(\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]-i\,\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]\right)}{b}\right)}{4\,a^{3/2}}+$$

$$\frac{-i\,\text{CoshIntegral}\left[-\frac{i\sqrt{a}\,d}{\sqrt{b}}+d\,x\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]+\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}-d\,x\right]}{2\,a^2}+$$

$$\left.\frac{i\sqrt{b}\left(-\frac{\text{Sinh}[d\,x]}{-i\sqrt{a}\sqrt{b}+b\,x}+\frac{d\left(\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]-i\,\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}-d\,x\right]\right)}{b}\right)}{4\,a^{3/2}}\right)-$$

$$\text{Sinh}[c]\left(\frac{\text{CoshIntegral}[d\,x]}{a^2}-\frac{i\sqrt{b}\left(-\frac{\text{Cosh}[d\,x]}{i\sqrt{a}\sqrt{b}+b\,x}+\frac{d\left(-i\,\text{CoshIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]+\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]\right)}{b}\right)}{4\,a^{3/2}}-\right.$$

$$\frac{\text{Cos}\left[\frac{\sqrt{a}\ d}{\sqrt{b}}\right]\ \text{CoshIntegral}\left[-\frac{i\sqrt{a}\ d}{\sqrt{b}}+d\ x\right]-i\ \text{Sin}\left[\frac{\sqrt{a}\ d}{\sqrt{b}}\right]\ \text{SinhIntegral}\left[\frac{i\sqrt{a}\ d}{\sqrt{b}}-d\ x\right]}{2\ a^2}+$$

$$\frac{i\sqrt{b}\left(-\frac{\text{Cosh}[d\ x]}{-i\sqrt{a}\ \sqrt{b}+b\ x}-\frac{d\left(-i\ \text{CoshIntegral}\left[d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]\ \text{Sin}\left[\frac{\sqrt{a}\ d}{\sqrt{b}}\right]+\text{Cos}\left[\frac{\sqrt{a}\ d}{\sqrt{b}}\right]\ \text{SinhIntegral}\left[\frac{i\sqrt{a}\ d}{\sqrt{b}}-d\ x\right]\right)}{b}\right)}{4\ a^{3/2}}-$$

$$\left.\left.\frac{\text{Cos}\left[\frac{\sqrt{a}\ d}{\sqrt{b}}\right]\ \text{CoshIntegral}\left[\frac{i\sqrt{a}\ d}{\sqrt{b}}+d\ x\right]-i\ \text{Sin}\left[\frac{\sqrt{a}\ d}{\sqrt{b}}\right]\ \text{SinhIntegral}\left[\frac{i\sqrt{a}\ d}{\sqrt{b}}+d\ x\right]}{2\ a^2}\right)\right)+$$

$$\frac{1}{2}\left(\text{Cosh}[c]\left(\frac{\text{SinhIntegral}[d\ x]}{a^2}-\frac{-i\ \text{CoshIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]\ \text{Sin}\left[\frac{\sqrt{a}\ d}{\sqrt{b}}\right]+\text{Cos}\left[\frac{\sqrt{a}\ d}{\sqrt{b}}\right]\ \text{SinhIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]}{2\ a^2}-\right.\right.$$

$$\frac{i\sqrt{b}\left(-\frac{\text{Sinh}[d\ x]}{i\sqrt{a}\ \sqrt{b}+b\ x}+\frac{d\left(\text{Cos}\left[\frac{\sqrt{a}\ d}{\sqrt{b}}\right]\ \text{CoshIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]-i\ \text{Sin}\left[\frac{\sqrt{a}\ d}{\sqrt{b}}\right]\ \text{SinhIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]\right)}{b}\right)}{4\ a^{3/2}}+$$

$$\frac{-i\ \text{CoshIntegral}\left[-\frac{i\sqrt{a}\ d}{\sqrt{b}}+d\ x\right]\ \text{Sin}\left[\frac{\sqrt{a}\ d}{\sqrt{b}}\right]+\text{Cos}\left[\frac{\sqrt{a}\ d}{\sqrt{b}}\right]\ \text{SinhIntegral}\left[\frac{i\sqrt{a}\ d}{\sqrt{b}}-d\ x\right]}{2\ a^2}+$$

$$\left.\frac{i\sqrt{b}\left(-\frac{\text{Sinh}[d\ x]}{-i\sqrt{a}\ \sqrt{b}+b\ x}+\frac{d\left(\text{Cos}\left[\frac{\sqrt{a}\ d}{\sqrt{b}}\right]\ \text{CoshIntegral}\left[d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]-i\ \text{Sin}\left[\frac{\sqrt{a}\ d}{\sqrt{b}}\right]\ \text{SinhIntegral}\left[\frac{i\sqrt{a}\ d}{\sqrt{b}}-d\ x\right]\right)}{b}\right)}{4\ a^{3/2}}\right)+$$

$$\text{Sinh}[c]\left(\frac{\text{CoshIntegral}[d\,x]}{a^2}-\frac{i\sqrt{b}\left(-\frac{\text{Cosh}[d\,x]}{i\sqrt{a}\sqrt{b}+b\,x}+\frac{d\left(-i\,\text{CoshIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]+\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]\right)}{b}\right)}{4\,a^{3/2}}-\right.$$

$$\frac{\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[-\frac{i\sqrt{a}\,d}{\sqrt{b}}+d\,x\right]-i\,\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}-d\,x\right]}{2\,a^2}+$$

$$\frac{i\sqrt{b}\left(-\frac{\text{Cosh}[d\,x]}{-i\sqrt{a}\sqrt{b}+b\,x}-\frac{d\left(-i\,\text{CoshIntegral}\left[d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]+\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}-d\,x\right]\right)}{b}\right)}{4\,a^{3/2}}-$$

$$\left.\frac{\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}+d\,x\right]-i\,\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}+d\,x\right]}{2\,a^2}\right)\Bigg)$$

## Problem 71: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{Cosh}[c+d\,x]}{x^2\left(a+b\,x^2\right)^2}\,dx$$

Optimal (type 4, 500 leaves, 32 steps):

$$-\frac{\text{Cosh}[c+d x]}{a^2 x}+\frac{\sqrt{b}\,\text{Cosh}[c+d x]}{4 a^2\left(\sqrt{-a}-\sqrt{b}\,x\right)}-\frac{\sqrt{b}\,\text{Cosh}[c+d x]}{4 a^2\left(\sqrt{-a}+\sqrt{b}\,x\right)}-\frac{3\sqrt{b}\,\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d x\right]}{4(-a)^{5/2}}+$$

$$\frac{3\sqrt{b}\,\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d x\right]}{4(-a)^{5/2}}+\frac{d\,\text{CoshIntegral}[d x]\,\text{Sinh}[c]}{a^2}+\frac{d\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d x\right]\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]}{4 a^2}+$$

$$\frac{d\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d x\right]\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]}{4 a^2}+\frac{d\,\text{Cosh}[c]\,\text{SinhIntegral}[d x]}{a^2}-\frac{d\,\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d x\right]}{4 a^2}+$$

$$\frac{3\sqrt{b}\,\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d x\right]}{4(-a)^{5/2}}+\frac{d\,\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d x\right]}{4 a^2}+\frac{3\sqrt{b}\,\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d x\right]}{4(-a)^{5/2}}$$

Result (type 4, 675 leaves) :

$$\frac{1}{4 a^{5/2} x\left(a+b x^2\right)}\left(-4 a^{3/2}\,\text{Cosh}[c+d x]-6\sqrt{a}\,b x^2\,\text{Cosh}[c+d x]+4 a^{3/2} d x\,\text{CoshIntegral}[d x]\,\text{Sinh}[c]+\right.$$

$$4\sqrt{a}\,b d x^3\,\text{CoshIntegral}[d x]\,\text{Sinh}[c]+x\left(a+b x^2\right)\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+i d x\right]\left(-3 i\sqrt{b}\,\text{Cosh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]+\sqrt{a}\,d\,\text{Sinh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\right)+$$

$$x\left(a+b x^2\right)\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i d x\right]\left(3 i\sqrt{b}\,\text{Cosh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]+\sqrt{a}\,d\,\text{Sinh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\right)+$$

$$4 a^{3/2} d x\,\text{Cosh}[c]\,\text{SinhIntegral}[d x]+4\sqrt{a}\,b d x^3\,\text{Cosh}[c]\,\text{SinhIntegral}[d x]+$$

$$i a^{3/2} d x\,\text{Cosh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i d x\right]+i\sqrt{a}\,b d x^3\,\text{Cosh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i d x\right]+$$

$$3 a\sqrt{b}\,x\,\text{Sinh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i d x\right]+3 b^{3/2} x^3\,\text{Sinh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i d x\right]-$$

$$i a^{3/2} d x\,\text{Cosh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i d x\right]-i\sqrt{a}\,b d x^3\,\text{Cosh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i d x\right]+$$

$$\left.3 a\sqrt{b}\,x\,\text{Sinh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i d x\right]+3 b^{3/2} x^3\,\text{Sinh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i d x\right]\right)$$

## Problem 72: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^3\,\text{Cosh}[c+d x]}{\left(a+b x^2\right)^3}\,dx$$

Optimal (type 4, 476 leaves, 27 steps) :

$$-\frac{x^2 \operatorname{Cosh}[c+d x]}{4 b \left(a+b x^2\right)^2}-\frac{\operatorname{Cosh}[c+d x]}{4 b^2 \left(a+b x^2\right)}+\frac{d^2 \operatorname{Cosh}\left[c+\frac{\sqrt{-a}\, d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}-d x\right]}{16 b^3}+\frac{d^2 \operatorname{Cosh}\left[c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}+d x\right]}{16 b^3}-$$

$$\frac{3 d \operatorname{CoshIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}+d x\right] \operatorname{Sinh}\left[c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right]}{16 \sqrt{-a}\, b^{5/2}}+\frac{3 d \operatorname{CoshIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}-d x\right] \operatorname{Sinh}\left[c+\frac{\sqrt{-a}\, d}{\sqrt{b}}\right]}{16 \sqrt{-a}\, b^{5/2}}-$$

$$\frac{d x \operatorname{Sinh}[c+d x]}{8 b^2 \left(a+b x^2\right)}-\frac{3 d \operatorname{Cosh}\left[c+\frac{\sqrt{-a}\, d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}-d x\right]}{16 \sqrt{-a}\, b^{5/2}}-\frac{d^2 \operatorname{Sinh}\left[c+\frac{\sqrt{-a}\, d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}-d x\right]}{16 b^3}-$$

$$\frac{3 d \operatorname{Cosh}\left[c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}+d x\right]}{16 \sqrt{-a}\, b^{5/2}}+\frac{d^2 \operatorname{Sinh}\left[c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a}\, d}{\sqrt{b}}+d x\right]}{16 b^3}$$

Result (type 4, 648 leaves):

$$\frac{1}{16 b^2}\left(-\frac{2 \operatorname{Cosh}[d x]\left(2\left(a+2 b x^2\right) \operatorname{Cosh}[c]+d x\left(a+b x^2\right) \operatorname{Sinh}[c]\right)}{\left(a+b x^2\right)^2}-\right.$$

$$\frac{2\left(d x\left(a+b x^2\right) \operatorname{Cosh}[c]+2\left(a+2 b x^2\right) \operatorname{Sinh}[c]\right) \operatorname{Sinh}[d x]}{\left(a+b x^2\right)^2}+\frac{1}{\sqrt{a}\sqrt{b}} 3\, \mathrm{i}\, d \operatorname{Sinh}[c]\left(\operatorname{Cos}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[-\frac{\sqrt{a}\, d}{\sqrt{b}}+\mathrm{i}\, d x\right]-\right.$$

$$\left.\operatorname{Cos}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}+\mathrm{i}\, d x\right]+\operatorname{Sin}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}\right]\left(\operatorname{SinIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}-\mathrm{i}\, d x\right]-\operatorname{SinIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}+\mathrm{i}\, d x\right]\right)\right)-$$

$$\frac{1}{b}\mathrm{i}\, d^2 \operatorname{Sinh}[c]\left(\operatorname{CosIntegral}\left[-\frac{\sqrt{a}\, d}{\sqrt{b}}+\mathrm{i}\, d x\right] \operatorname{Sin}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}\right]-\operatorname{CosIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}+\mathrm{i}\, d x\right] \operatorname{Sin}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}\right]+\right.$$

$$\left.\operatorname{Cos}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}\right]\left(-\operatorname{SinIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}-\mathrm{i}\, d x\right]+\operatorname{SinIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}+\mathrm{i}\, d x\right]\right)\right)+$$

$$\frac{1}{\sqrt{a}\sqrt{b}} 3 d \operatorname{Cosh}[c]\left(\operatorname{CosIntegral}\left[-\frac{\sqrt{a}\, d}{\sqrt{b}}+\mathrm{i}\, d x\right] \operatorname{Sin}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}\right]+\operatorname{CosIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}+\mathrm{i}\, d x\right] \operatorname{Sin}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}\right]-\right.$$

$$\left.\operatorname{Cos}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}\right]\left(\operatorname{SinIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}-\mathrm{i}\, d x\right]+\operatorname{SinIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}+\mathrm{i}\, d x\right]\right)\right)+$$

$$\frac{1}{b} d^2 \operatorname{Cosh}[c]\left(\operatorname{Cos}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[-\frac{\sqrt{a}\, d}{\sqrt{b}}+\mathrm{i}\, d x\right]+\operatorname{Cos}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}+\mathrm{i}\, d x\right]+\right.$$

$$\left.\left.\operatorname{Sin}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}\right]\left(\operatorname{SinIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}-\mathrm{i}\, d x\right]+\operatorname{SinIntegral}\left[\frac{\sqrt{a}\, d}{\sqrt{b}}+\mathrm{i}\, d x\right]\right)\right)\right)$$

## Problem 73: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 \operatorname{Cosh}[c+d x]}{\left(a+b x^2\right)^3} \, dx$$

Optimal (type 4, 746 leaves, 28 steps):

$$-\frac{\operatorname{Cosh}[c+d x]}{16 a b^{3/2}\left(\sqrt{-a}-\sqrt{b} x\right)}+\frac{\operatorname{Cosh}[c+d x]}{16 a b^{3/2}\left(\sqrt{-a}+\sqrt{b} x\right)}-\frac{x \operatorname{Cosh}[c+d x]}{4 b\left(a+b x^2\right)^2}-\frac{\operatorname{Cosh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{16(-a)^{3/2} b^{3/2}}+$$
$$\frac{d^2 \operatorname{Cosh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{16 \sqrt{-a} b^{5/2}}+\frac{\operatorname{Cosh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{16(-a)^{3/2} b^{3/2}}-\frac{d^2 \operatorname{Cosh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{16 \sqrt{-a} b^{5/2}}-$$
$$\frac{d \operatorname{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right] \operatorname{Sinh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 a b^2}-\frac{d \operatorname{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right] \operatorname{Sinh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 a b^2}-\frac{d \operatorname{Sinh}[c+d x]}{8 b^2\left(a+b x^2\right)}+$$
$$\frac{d \operatorname{Cosh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{16 a b^2}+\frac{\operatorname{Sinh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{16(-a)^{3/2} b^{3/2}}-\frac{d^2 \operatorname{Sinh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{16 \sqrt{-a} b^{5/2}}-$$
$$\frac{d \operatorname{Cosh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{16 a b^2}+\frac{\operatorname{Sinh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{16(-a)^{3/2} b^{3/2}}-\frac{d^2 \operatorname{Sinh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{16 \sqrt{-a} b^{5/2}}$$

Result (type 4, 932 leaves):

$$\frac{1}{16\,a^{3/2}\,b^2}\left(-\frac{2\,a^{3/2}\,b\,x\,\text{Cosh}[c]\,\text{Cosh}[d\,x]}{(a+b\,x^2)^2}+\frac{2\,\sqrt{a}\,b^2\,x^3\,\text{Cosh}[c]\,\text{Cosh}[d\,x]}{(a+b\,x^2)^2}-\frac{2\,a^{5/2}\,d\,\text{Cosh}[d\,x]\,\text{Sinh}[c]}{(a+b\,x^2)^2}-\right.$$

$$\frac{2\,a^{3/2}\,b\,d\,x^2\,\text{Cosh}[d\,x]\,\text{Sinh}[c]}{(a+b\,x^2)^2}+\frac{\mathrm{i}\,\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]\left((b+a\,d^2)\,\text{Cosh}\left[c-\frac{\mathrm{i}\,\sqrt{a}\,d}{\sqrt{b}}\right]+\mathrm{i}\,\sqrt{a}\,\sqrt{b}\,d\,\text{Sinh}\left[c-\frac{\mathrm{i}\,\sqrt{a}\,d}{\sqrt{b}}\right]\right)}{\sqrt{b}}-$$

$$\frac{\mathrm{i}\,\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]\left((b+a\,d^2)\,\text{Cosh}\left[c+\frac{\mathrm{i}\,\sqrt{a}\,d}{\sqrt{b}}\right]-\mathrm{i}\,\sqrt{a}\,\sqrt{b}\,d\,\text{Sinh}\left[c+\frac{\mathrm{i}\,\sqrt{a}\,d}{\sqrt{b}}\right]\right)}{\sqrt{b}}-\frac{2\,a^{5/2}\,d\,\text{Cosh}[c]\,\text{Sinh}[d\,x]}{(a+b\,x^2)^2}-$$

$$\frac{2\,a^{3/2}\,b\,d\,x^2\,\text{Cosh}[c]\,\text{Sinh}[d\,x]}{(a+b\,x^2)^2}-\frac{2\,a^{3/2}\,b\,x\,\text{Sinh}[c]\,\text{Sinh}[d\,x]}{(a+b\,x^2)^2}+\frac{2\,\sqrt{a}\,b^2\,x^3\,\text{Sinh}[c]\,\text{Sinh}[d\,x]}{(a+b\,x^2)^2}-$$

$$\mathrm{i}\,\sqrt{a}\,d\,\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{Cosh}[c]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-\mathrm{i}\,d\,x\right]+\mathrm{i}\,\sqrt{b}\,\text{Cosh}[c]\,\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-\mathrm{i}\,d\,x\right]+$$

$$\frac{\mathrm{i}\,a\,d^2\,\text{Cosh}[c]\,\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-\mathrm{i}\,d\,x\right]}{\sqrt{b}}-\sqrt{b}\,\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{Sinh}[c]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-\mathrm{i}\,d\,x\right]-$$

$$\frac{a\,d^2\,\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{Sinh}[c]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-\mathrm{i}\,d\,x\right]}{\sqrt{b}}-\sqrt{a}\,d\,\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{Sinh}[c]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-\mathrm{i}\,d\,x\right]+$$

$$\mathrm{i}\,\sqrt{a}\,d\,\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{Cosh}[c]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]-\mathrm{i}\,\sqrt{b}\,\text{Cosh}[c]\,\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]-$$

$$\frac{\mathrm{i}\,a\,d^2\,\text{Cosh}[c]\,\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]}{\sqrt{b}}-\sqrt{b}\,\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{Sinh}[c]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]-$$

$$\left.\frac{a\,d^2\,\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{Sinh}[c]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]}{\sqrt{b}}-\sqrt{a}\,d\,\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{Sinh}[c]\,\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\mathrm{i}\,d\,x\right]\right)$$

## Problem 74: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\,\text{Cosh}[c+d\,x]}{(a+b\,x^2)^3}\,dx$$

Optimal (type 4, 512 leaves, 19 steps):

$$-\frac{\text{Cosh}[c+d\,x]}{4\,b\,\left(a+b\,x^2\right)^2}-\frac{d^2\,\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{16\,a\,b^2}-\frac{d^2\,\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{16\,a\,b^2}+$$

$$\frac{d\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]\,\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]}{16\,(-a)^{3/2}\,b^{3/2}}-\frac{d\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]\,\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]}{16\,(-a)^{3/2}\,b^{3/2}}-\frac{d\,\text{Sinh}[c+d\,x]}{16\,a\,b^{3/2}\,\left(\sqrt{-a}-\sqrt{b}\,x\right)}+$$

$$\frac{d\,\text{Sinh}[c+d\,x]}{16\,a\,b^{3/2}\,\left(\sqrt{-a}+\sqrt{b}\,x\right)}+\frac{d\,\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{16\,(-a)^{3/2}\,b^{3/2}}+\frac{d^2\,\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{16\,a\,b^2}+$$

$$\frac{d\,\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{16\,(-a)^{3/2}\,b^{3/2}}-\frac{d^2\,\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{16\,a\,b^2}$$

Result (type 4, 637 leaves):

$$\frac{1}{16\,a\,b}\left(\frac{2\,\text{Cosh}[d\,x]\,\left(-2\,a\,\text{Cosh}[c]+d\,x\,\left(a+b\,x^2\right)\,\text{Sinh}[c]\right)}{\left(a+b\,x^2\right)^2}+\right.$$

$$\frac{2\,\left(d\,x\,\left(a+b\,x^2\right)\,\text{Cosh}[c]-2\,a\,\text{Sinh}[c]\right)\,\text{Sinh}[d\,x]}{\left(a+b\,x^2\right)^2}+\frac{1}{\sqrt{a}\,\sqrt{b}}\,\text{i}\,d\,\text{Sinh}[c]\,\left(\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+\text{i}\,d\,x\right]-\right.$$

$$\left.\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\text{i}\,d\,x\right]+\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\left(\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-\text{i}\,d\,x\right]-\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\text{i}\,d\,x\right]\right)\right)+$$

$$\frac{1}{b}\,\text{i}\,d^2\,\text{Sinh}[c]\,\left(\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+\text{i}\,d\,x\right]\,\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]-\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\text{i}\,d\,x\right]\,\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]+\right.$$

$$\left.\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\left(-\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-\text{i}\,d\,x\right]+\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\text{i}\,d\,x\right]\right)\right)+$$

$$\frac{1}{\sqrt{a}\,\sqrt{b}}\,d\,\text{Cosh}[c]\,\left(\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+\text{i}\,d\,x\right]\,\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]+\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\text{i}\,d\,x\right]\,\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]-\right.$$

$$\left.\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\left(\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-\text{i}\,d\,x\right]+\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\text{i}\,d\,x\right]\right)\right)-$$

$$\frac{1}{b}\,d^2\,\text{Cosh}[c]\,\left(\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+\text{i}\,d\,x\right]+\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\text{i}\,d\,x\right]+\right.$$

$$\left.\left.\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\left(\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-\text{i}\,d\,x\right]+\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+\text{i}\,d\,x\right]\right)\right)\right)$$

## Problem 75: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Cosh}[c+d\,x]}{\left(a+b\,x^2\right)^3}\,dx$$

Optimal (type 4, 856 leaves, 28 steps):

$$-\frac{\text{Cosh}[c+d\,x]}{16\,(-a)^{3/2}\sqrt{b}\,\left(\sqrt{-a}-\sqrt{b}\,x\right)^2}-\frac{3\,\text{Cosh}[c+d\,x]}{16\,a^2\sqrt{b}\,\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\text{Cosh}[c+d\,x]}{16\,(-a)^{3/2}\sqrt{b}\,\left(\sqrt{-a}+\sqrt{b}\,x\right)^2}+$$

$$\frac{3\,\text{Cosh}[c+d\,x]}{16\,a^2\sqrt{b}\,\left(\sqrt{-a}+\sqrt{b}\,x\right)}+\frac{3\,\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{16\,(-a)^{5/2}\sqrt{b}}+\frac{d^2\,\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{16\,(-a)^{3/2}\,b^{3/2}}-$$

$$\frac{3\,\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{16\,(-a)^{5/2}\sqrt{b}}-\frac{d^2\,\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{16\,(-a)^{3/2}\,b^{3/2}}-\frac{3\,d\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]}{16\,a^2\,b}-$$

$$\frac{3\,d\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]}{16\,a^2\,b}+\frac{d\,\text{Sinh}[c+d\,x]}{16\,(-a)^{3/2}\,b\,\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{d\,\text{Sinh}[c+d\,x]}{16\,(-a)^{3/2}\,b\,\left(\sqrt{-a}+\sqrt{b}\,x\right)}+$$

$$\frac{3\,d\,\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{16\,a^2\,b}-\frac{3\,\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{16\,(-a)^{5/2}\sqrt{b}}-\frac{d^2\,\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{16\,(-a)^{3/2}\,b^{3/2}}-$$

$$\frac{3\,d\,\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{16\,a^2\,b}-\frac{3\,\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{16\,(-a)^{5/2}\sqrt{b}}-\frac{d^2\,\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{16\,(-a)^{3/2}\,b^{3/2}}$$

Result (type 4, 933 leaves):

$$\frac{1}{16\,a^2\,b^{3/2}}\left(\frac{10\,a\,b^{3/2}\,x\,\mathrm{Cosh}[c]\,\mathrm{Cosh}[d\,x]}{\left(a+b\,x^2\right)^2}+\frac{6\,b^{5/2}\,x^3\,\mathrm{Cosh}[c]\,\mathrm{Cosh}[d\,x]}{\left(a+b\,x^2\right)^2}+\frac{2\,a^2\,\sqrt{b}\,d\,\mathrm{Cosh}[d\,x]\,\mathrm{Sinh}[c]}{\left(a+b\,x^2\right)^2}+\right.$$

$$\frac{2\,a\,b^{3/2}\,d\,x^2\,\mathrm{Cosh}[d\,x]\,\mathrm{Sinh}[c]}{\left(a+b\,x^2\right)^2}+\frac{\mathrm{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\left(i\,\left(3\,b-a\,d^2\right)\,\mathrm{Cosh}\left[c-\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]-3\,\sqrt{a}\,\sqrt{b}\,d\,\mathrm{Sinh}\left[c-\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]\right)}{\sqrt{a}}+$$

$$\frac{i\,\mathrm{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\left(\left(-3\,b+a\,d^2\right)\,\mathrm{Cosh}\left[c+\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]+3\,i\,\sqrt{a}\,\sqrt{b}\,d\,\mathrm{Sinh}\left[c+\frac{i\,\sqrt{a}\,d}{\sqrt{b}}\right]\right)}{\sqrt{a}}+$$

$$\frac{2\,a^2\,\sqrt{b}\,d\,\mathrm{Cosh}[c]\,\mathrm{Sinh}[d\,x]}{\left(a+b\,x^2\right)^2}+\frac{2\,a\,b^{3/2}\,d\,x^2\,\mathrm{Cosh}[c]\,\mathrm{Sinh}[d\,x]}{\left(a+b\,x^2\right)^2}+\frac{10\,a\,b^{3/2}\,x\,\mathrm{Sinh}[c]\,\mathrm{Sinh}[d\,x]}{\left(a+b\,x^2\right)^2}+\frac{6\,b^{5/2}\,x^3\,\mathrm{Sinh}[c]\,\mathrm{Sinh}[d\,x]}{\left(a+b\,x^2\right)^2}-$$

$$3\,i\,\sqrt{b}\,d\,\mathrm{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\mathrm{Cosh}[c]\,\mathrm{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i\,d\,x\right]+\frac{3\,i\,b\,\mathrm{Cosh}[c]\,\mathrm{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\mathrm{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i\,d\,x\right]}{\sqrt{a}}-$$

$$i\,\sqrt{a}\,d^2\,\mathrm{Cosh}[c]\,\mathrm{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\mathrm{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i\,d\,x\right]-\frac{3\,b\,\mathrm{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\mathrm{Sinh}[c]\,\mathrm{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i\,d\,x\right]}{\sqrt{a}}+$$

$$\sqrt{a}\,d^2\,\mathrm{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\mathrm{Sinh}[c]\,\mathrm{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i\,d\,x\right]-3\,\sqrt{b}\,d\,\mathrm{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\mathrm{Sinh}[c]\,\mathrm{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i\,d\,x\right]+$$

$$3\,i\,\sqrt{b}\,d\,\mathrm{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\mathrm{Cosh}[c]\,\mathrm{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]-\frac{3\,i\,b\,\mathrm{Cosh}[c]\,\mathrm{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\mathrm{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]}{\sqrt{a}}+$$

$$i\,\sqrt{a}\,d^2\,\mathrm{Cosh}[c]\,\mathrm{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\mathrm{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]-\frac{3\,b\,\mathrm{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\mathrm{Sinh}[c]\,\mathrm{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]}{\sqrt{a}}+$$

$$\left.\sqrt{a}\,d^2\,\mathrm{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\mathrm{Sinh}[c]\,\mathrm{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]-3\,\sqrt{b}\,d\,\mathrm{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\,\mathrm{Sinh}[c]\,\mathrm{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\right)$$

## Problem 76: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\mathrm{Cosh}[c+d\,x]}{x\,\left(a+b\,x^2\right)^3}\,dx$$

Optimal (type 4, 730 leaves, 41 steps) :

$$\frac{\text{Cosh}[c+d x]}{4 a \left(a+b x^2\right)^2}+\frac{\text{Cosh}[c+d x]}{2 a^2 \left(a+b x^2\right)}+\frac{\text{Cosh}[c] \text{CoshIntegral}[d x]}{a^3}-\frac{\text{Cosh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{2 a^3}+$$

$$\frac{d^2 \text{Cosh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{16 a^2 b}-\frac{\text{Cosh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{2 a^3}+\frac{d^2 \text{Cosh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{16 a^2 b}+$$

$$\frac{5 d \text{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right] \text{Sinh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 (-a)^{5/2} \sqrt{b}}-\frac{5 d \text{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right] \text{Sinh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 (-a)^{5/2} \sqrt{b}}+\frac{d \text{Sinh}[c+d x]}{16 a^2 \sqrt{b} \left(\sqrt{-a}-\sqrt{b} x\right)}-$$

$$\frac{d \text{Sinh}[c+d x]}{16 a^2 \sqrt{b} \left(\sqrt{-a}+\sqrt{b} x\right)}+\frac{\text{Sinh}[c] \text{SinhIntegral}[d x]}{a^3}+\frac{5 d \text{Cosh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{16 (-a)^{5/2} \sqrt{b}}+$$

$$\frac{\text{Sinh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{2 a^3}-\frac{d^2 \text{Sinh}\left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{16 a^2 b}+$$

$$\frac{5 d \text{Cosh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{16 (-a)^{5/2} \sqrt{b}}-\frac{\text{Sinh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{2 a^3}+\frac{d^2 \text{Sinh}\left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{16 a^2 b}$$

Result (type 4, 1558 leaves) :

$$\frac{1}{16 a^3 b \left(a+b x^2\right)^2}$$

$$\Bigg(12 a^2 b \text{Cosh}[c+d x]+8 a b^2 x^2 \text{Cosh}[c+d x]+16 b \left(a+b x^2\right)^2 \text{Cosh}[c] \text{CoshIntegral}[d x]-8 a^2 b \text{Cosh}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{CoshIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right]+$$

$$a^3 d^2 \text{Cosh}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{CoshIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right]-16 a b^2 x^2 \text{Cosh}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{CoshIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right]+$$

$$2 a^2 b d^2 x^2 \text{Cosh}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{CoshIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right]-8 b^3 x^4 \text{Cosh}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{CoshIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right]+$$

$$a b^2 d^2 x^4 \text{Cosh}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{CoshIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right]-5 i a^{5/2} \sqrt{b} d \text{CoshIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right] \text{Sinh}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right]-$$

$$10 i a^{3/2} b^{3/2} d x^2 \text{CoshIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right] \text{Sinh}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right]-5 i \sqrt{a} b^{5/2} d x^4 \text{CoshIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right] \text{Sinh}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right]+$$

$$\left(a+b x^2\right)^2 \text{CoshIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right] \left(\left(-8 b+a d^2\right) \text{Cosh}\left[c+\frac{i \sqrt{a} d}{\sqrt{b}}\right]+5 i \sqrt{a} \sqrt{b} d \text{Sinh}\left[c+\frac{i \sqrt{a} d}{\sqrt{b}}\right]\right)-2 a^2 b d x \text{Sinh}[c+d x]-$$

$$2 a b^2 d x^3 \text{Sinh}[c+d x]+16 a^2 b \text{Sinh}[c] \text{SinhIntegral}[d x]+32 a b^2 x^2 \text{Sinh}[c] \text{SinhIntegral}[d x]+16 b^3 x^4 \text{Sinh}[c] \text{SinhIntegral}[d x]-$$

$$5\,i\,a^{5/2}\sqrt{b}\,d\,\text{Cosh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]-10\,i\,a^{3/2}b^{3/2}d\,x^2\,\text{Cosh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]-$$

$$5\,i\sqrt{a}\,b^{5/2}d\,x^4\,\text{Cosh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]-8\,a^2 b\,\text{Sinh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]+$$

$$a^3 d^2\,\text{Sinh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]-16\,a\,b^2x^2\,\text{Sinh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]+$$

$$2\,a^2 b\,d^2x^2\,\text{Sinh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]-8\,b^3x^4\,\text{Sinh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]+$$

$$a\,b^2d^2x^4\,\text{Sinh}\left[c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]-5\,i\,a^{5/2}\sqrt{b}\,d\,\text{Cosh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}-d\,x\right]-$$

$$10\,i\,a^{3/2}b^{3/2}d\,x^2\,\text{Cosh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}-d\,x\right]-5\,i\sqrt{a}\,b^{5/2}d\,x^4\,\text{Cosh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}-d\,x\right]+$$

$$8\,a^2 b\,\text{Sinh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}-d\,x\right]-a^3d^2\,\text{Sinh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}-d\,x\right]+$$

$$16\,a\,b^2x^2\,\text{Sinh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}-d\,x\right]-2\,a^2b\,d^2x^2\,\text{Sinh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}-d\,x\right]+$$

$$8\,b^3x^4\,\text{Sinh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}-d\,x\right]-a\,b^2d^2x^4\,\text{Sinh}\left[c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{i\sqrt{a}\,d}{\sqrt{b}}-d\,x\right]\Bigg)$$

## Problem 77: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{Cosh}[c+d\,x]}{x^2\left(a+b\,x^2\right)^3}\,dx$$

Optimal (type 4, 874 leaves, 60 steps):

$$-\frac{\text{Cosh}[c+d\,x]}{a^3\,x}-\frac{\sqrt{b}\,\text{Cosh}[c+d\,x]}{16\,(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)^2}+\frac{7\sqrt{b}\,\text{Cosh}[c+d\,x]}{16\,a^3\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\sqrt{b}\,\text{Cosh}[c+d\,x]}{16\,(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)^2}-$$

$$\frac{7\sqrt{b}\,\text{Cosh}[c+d\,x]}{16\,a^3\left(\sqrt{-a}+\sqrt{b}\,x\right)}+\frac{15\sqrt{b}\,\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{16\,(-a)^{7/2}}+\frac{d^2\,\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{16\,(-a)^{5/2}\sqrt{b}}-$$

$$\frac{15\sqrt{b}\,\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{16\,(-a)^{7/2}}-\frac{d^2\,\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{16\,(-a)^{5/2}\sqrt{b}}+\frac{d\,\text{CoshIntegral}[d\,x]\,\text{Sinh}[c]}{a^3}+$$

$$\frac{7\,d\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]}{16\,a^3}+\frac{7\,d\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]}{16\,a^3}+\frac{d\,\text{Sinh}[c+d\,x]}{16\,(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)}+$$

$$\frac{d\,\text{Sinh}[c+d\,x]}{16\,(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)}+\frac{d\,\text{Cosh}[c]\,\text{SinhIntegral}[d\,x]}{a^3}-\frac{7\,d\,\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{16\,a^3}-$$

$$\frac{15\sqrt{b}\,\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{16\,(-a)^{7/2}}-\frac{d^2\,\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{16\,(-a)^{5/2}\sqrt{b}}+$$

$$\frac{7\,d\,\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{16\,a^3}-\frac{15\sqrt{b}\,\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{16\,(-a)^{7/2}}-\frac{d^2\,\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{16\,(-a)^{5/2}\sqrt{b}}$$

Result (type 4, 1359 leaves) :

$$\frac{1}{16 a^{7/2} \sqrt{b} x (a + b x^2)^2} \Big(-16 a^{5/2} \sqrt{b} \text{Cosh}[c + d x] - 50 a^{3/2} b^{3/2} x^2 \text{Cosh}[c + d x] - 30 \sqrt{a} b^{5/2} x^4 \text{Cosh}[c + d x] +$$
$$16 a^{5/2} \sqrt{b} d x \text{CoshIntegral}[d x] \text{Sinh}[c] + 32 a^{3/2} b^{3/2} d x^3 \text{CoshIntegral}[d x] \text{Sinh}[c] + 16 \sqrt{a} b^{5/2} d x^5 \text{CoshIntegral}[d x] \text{Sinh}[c] +$$
$$x (a + b x^2)^2 \text{CoshIntegral}\Big[d \Big(\frac{i \sqrt{a}}{\sqrt{b}} + x\Big)\Big] \Big(-i (15 b - a d^2) \text{Cosh}\Big[c - \frac{i \sqrt{a} d}{\sqrt{b}}\Big] + 7 \sqrt{a} \sqrt{b} d \text{Sinh}\Big[c - \frac{i \sqrt{a} d}{\sqrt{b}}\Big]\Big) +$$
$$x (a + b x^2)^2 \text{CoshIntegral}\Big[d \Big(-\frac{i \sqrt{a}}{\sqrt{b}} + x\Big)\Big] \Big(i (15 b - a d^2) \text{Cosh}\Big[c + \frac{i \sqrt{a} d}{\sqrt{b}}\Big] + 7 \sqrt{a} \sqrt{b} d \text{Sinh}\Big[c + \frac{i \sqrt{a} d}{\sqrt{b}}\Big]\Big) -$$
$$2 a^{5/2} \sqrt{b} d x \text{Sinh}[c + d x] - 2 a^{3/2} b^{3/2} d x^3 \text{Sinh}[c + d x] + 16 a^{5/2} \sqrt{b} d x \text{Cosh}[c] \text{SinhIntegral}[d x] +$$
$$32 a^{3/2} b^{3/2} d x^3 \text{Cosh}[c] \text{SinhIntegral}[d x] + 16 \sqrt{a} b^{5/2} d x^5 \text{Cosh}[c] \text{SinhIntegral}[d x] +$$
$$7 a^{5/2} \sqrt{b} d x \text{Cosh}\Big[c - \frac{i \sqrt{a} d}{\sqrt{b}}\Big] \text{SinhIntegral}\Big[d \Big(\frac{i \sqrt{a}}{\sqrt{b}} + x\Big)\Big] + 14 a^{3/2} b^{3/2} d x^3 \text{Cosh}\Big[c - \frac{i \sqrt{a} d}{\sqrt{b}}\Big] \text{SinhIntegral}\Big[d \Big(\frac{i \sqrt{a}}{\sqrt{b}} + x\Big)\Big] +$$
$$7 \sqrt{a} b^{5/2} d x^5 \text{Cosh}\Big[c - \frac{i \sqrt{a} d}{\sqrt{b}}\Big] \text{SinhIntegral}\Big[d \Big(\frac{i \sqrt{a}}{\sqrt{b}} + x\Big)\Big] - 15 i a^2 b x \text{Sinh}\Big[c - \frac{i \sqrt{a} d}{\sqrt{b}}\Big] \text{SinhIntegral}\Big[d \Big(\frac{i \sqrt{a}}{\sqrt{b}} + x\Big)\Big] +$$
$$i a^3 d^2 x \text{Sinh}\Big[c - \frac{i \sqrt{a} d}{\sqrt{b}}\Big] \text{SinhIntegral}\Big[d \Big(\frac{i \sqrt{a}}{\sqrt{b}} + x\Big)\Big] - 30 i a b^2 x^3 \text{Sinh}\Big[c - \frac{i \sqrt{a} d}{\sqrt{b}}\Big] \text{SinhIntegral}\Big[d \Big(\frac{i \sqrt{a}}{\sqrt{b}} + x\Big)\Big] +$$
$$2 i a^2 b d^2 x^3 \text{Sinh}\Big[c - \frac{i \sqrt{a} d}{\sqrt{b}}\Big] \text{SinhIntegral}\Big[d \Big(\frac{i \sqrt{a}}{\sqrt{b}} + x\Big)\Big] - 15 i b^3 x^5 \text{Sinh}\Big[c - \frac{i \sqrt{a} d}{\sqrt{b}}\Big] \text{SinhIntegral}\Big[d \Big(\frac{i \sqrt{a}}{\sqrt{b}} + x\Big)\Big] +$$
$$i a b^2 d^2 x^5 \text{Sinh}\Big[c - \frac{i \sqrt{a} d}{\sqrt{b}}\Big] \text{SinhIntegral}\Big[d \Big(\frac{i \sqrt{a}}{\sqrt{b}} + x\Big)\Big] - 7 a^{5/2} \sqrt{b} d x \text{Cosh}\Big[c + \frac{i \sqrt{a} d}{\sqrt{b}}\Big] \text{SinhIntegral}\Big[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\Big] -$$
$$14 a^{3/2} b^{3/2} d x^3 \text{Cosh}\Big[c + \frac{i \sqrt{a} d}{\sqrt{b}}\Big] \text{SinhIntegral}\Big[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\Big] - 7 \sqrt{a} b^{5/2} d x^5 \text{Cosh}\Big[c + \frac{i \sqrt{a} d}{\sqrt{b}}\Big] \text{SinhIntegral}\Big[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\Big] -$$
$$15 i a^2 b x \text{Sinh}\Big[c + \frac{i \sqrt{a} d}{\sqrt{b}}\Big] \text{SinhIntegral}\Big[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\Big] + i a^3 d^2 x \text{Sinh}\Big[c + \frac{i \sqrt{a} d}{\sqrt{b}}\Big] \text{SinhIntegral}\Big[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\Big] -$$
$$30 i a b^2 x^3 \text{Sinh}\Big[c + \frac{i \sqrt{a} d}{\sqrt{b}}\Big] \text{SinhIntegral}\Big[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\Big] + 2 i a^2 b d^2 x^3 \text{Sinh}\Big[c + \frac{i \sqrt{a} d}{\sqrt{b}}\Big] \text{SinhIntegral}\Big[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\Big] -$$
$$15 i b^3 x^5 \text{Sinh}\Big[c + \frac{i \sqrt{a} d}{\sqrt{b}}\Big] \text{SinhIntegral}\Big[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\Big] + i a b^2 d^2 x^5 \text{Sinh}\Big[c + \frac{i \sqrt{a} d}{\sqrt{b}}\Big] \text{SinhIntegral}\Big[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\Big]\Big)$$

## Problem 78: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Cosh}[c + d x]}{x^3 (a + b x^2)^3} \, dx$$

Optimal (type 4, 791 leaves, 46 steps):

$$-\frac{\text{Cosh}[c+d\,x]}{2\,a^3\,x^2}-\frac{b\,\text{Cosh}[c+d\,x]}{4\,a^2\,\left(a+b\,x^2\right)^2}-\frac{b\,\text{Cosh}[c+d\,x]}{a^3\,\left(a+b\,x^2\right)}-\frac{3\,b\,\text{Cosh}[c]\,\text{CoshIntegral}[d\,x]}{a^4}+\frac{d^2\,\text{Cosh}[c]\,\text{CoshIntegral}[d\,x]}{2\,a^3}+$$

$$\frac{3\,b\,\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{2\,a^4}-\frac{d^2\,\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{16\,a^3}+\frac{3\,b\,\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{2\,a^4}-$$

$$\frac{d^2\,\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{16\,a^3}+\frac{9\,\sqrt{b}\,d\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]\,\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]}{16\,(-a)^{7/2}}-$$

$$\frac{9\,\sqrt{b}\,d\,\text{CoshIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]\,\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]}{16\,(-a)^{7/2}}-\frac{d\,\text{Sinh}[c+d\,x]}{2\,a^3\,x}-\frac{\sqrt{b}\,d\,\text{Sinh}[c+d\,x]}{16\,a^3\,\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\sqrt{b}\,d\,\text{Sinh}[c+d\,x]}{16\,a^3\,\left(\sqrt{-a}+\sqrt{b}\,x\right)}-$$

$$\frac{3\,b\,\text{Sinh}[c]\,\text{SinhIntegral}[d\,x]}{a^4}+\frac{d^2\,\text{Sinh}[c]\,\text{SinhIntegral}[d\,x]}{2\,a^3}+\frac{9\,\sqrt{b}\,d\,\text{Cosh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{16\,(-a)^{7/2}}-$$

$$\frac{3\,b\,\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{2\,a^4}+\frac{d^2\,\text{Sinh}\left[c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\,x\right]}{16\,a^3}+$$

$$\frac{9\,\sqrt{b}\,d\,\text{Cosh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{16\,(-a)^{7/2}}+\frac{3\,b\,\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{2\,a^4}-\frac{d^2\,\text{Sinh}\left[c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right]\,\text{SinhIntegral}\left[\frac{\sqrt{-a}\,d}{\sqrt{b}}+d\,x\right]}{16\,a^3}$$

Result (type 4, 998 leaves):

$$-\frac{1}{16\,a^4}\left(\frac{2\,a\,\text{Cosh}[d\,x]\,\left(2\,\left(2\,a^2+9\,a\,b\,x^2+6\,b^2\,x^4\right)\text{Cosh}[c]+d\,x\,\left(4\,a^2+7\,a\,b\,x^2+3\,b^2\,x^4\right)\text{Sinh}[c]\right)}{x^2\,\left(a+b\,x^2\right)^2}+\right.$$

$$\frac{2\,a\,\left(d\,x\,\left(4\,a^2+7\,a\,b\,x^2+3\,b^2\,x^4\right)\text{Cosh}[c]+2\,\left(2\,a^2+9\,a\,b\,x^2+6\,b^2\,x^4\right)\text{Sinh}[c]\right)\text{Sinh}[d\,x]}{x^2\,\left(a+b\,x^2\right)^2}+$$

$$8\,\left(6\,b-a\,d^2\right)\,\left(\text{Cosh}[c]\,\text{CoshIntegral}[d\,x]+\text{Sinh}[c]\,\text{SinhIntegral}[d\,x]\right)-$$

$$9\,i\,\sqrt{a}\,\sqrt{b}\,d\,\text{Sinh}[c]\,\left(\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]-\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]+\right.$$

$$\left.\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\left(\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i\,d\,x\right]-\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\right)\right)+24\,i\,b\,\text{Sinh}[c]\,\left(\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]-\right.$$

$$\left.\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]+\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\left(-\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i\,d\,x\right]+\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\right)\right)-$$

$$i\,a\,d^2\,\text{Sinh}[c]\,\left(\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]-\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]+\right.$$

$$\left.\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\left(-\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i\,d\,x\right]+\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\right)\right)-$$

$$9\,\sqrt{a}\,\sqrt{b}\,d\,\text{Cosh}[c]\,\left(\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]+\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]-\right.$$

$$\left.\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\left(\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i\,d\,x\right]+\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\right)\right)-$$

$$24\,b\,\text{Cosh}[c]\,\left(\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]+\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]+\right.$$

$$\left.\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\left(\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i\,d\,x\right]+\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\right)\right)+$$

$$a\,d^2\,\text{Cosh}[c]\,\left(\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{CosIntegral}\left[-\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]+\text{Cos}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\text{CosIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]+\right.$$

$$\left.\left.\text{Sin}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}\right]\left(\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}-i\,d\,x\right]+\text{SinIntegral}\left[\frac{\sqrt{a}\,d}{\sqrt{b}}+i\,d\,x\right]\right)\right)\right)$$

## Problem 94: Result is not expressed in closed-form.

$$\int\frac{x^4\,\text{Cosh}[c+d\,x]}{a+b\,x^3}\,dx$$

Optimal (type 4, 373 leaves, 15 steps):

$$-\frac{\text{Cosh}[c+d\,x]}{b\,d^2}+\frac{(-1)^{2/3}a^{2/3}\,\text{Cosh}\left[c+\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,b^{5/3}}-\frac{(-1)^{1/3}a^{2/3}\,\text{Cosh}\left[c-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]\text{CoshIntegral}\left[-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,b^{5/3}}+\frac{a^{2/3}\,\text{Cosh}\left[c-\frac{a^{1/3}d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,b^{5/3}}+\frac{x\,\text{Sinh}[c+d\,x]}{b\,d}-\frac{(-1)^{2/3}a^{2/3}\,\text{Sinh}\left[c+\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,b^{5/3}}+\frac{a^{2/3}\,\text{Sinh}\left[c-\frac{a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,b^{5/3}}-\frac{(-1)^{1/3}a^{2/3}\,\text{Sinh}\left[c-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,b^{5/3}}$$

Result (type 7, 213 leaves):

$$-\frac{1}{6\,b^2\,d^2}\left(a\,d^2\,\text{RootSum}\left[a+b\,\#1^3\,\&,\ \frac{1}{\#1}\left(\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}\left[d\,(x-\#1)\right]-\text{CoshIntegral}\left[d\,(x-\#1)\right]\text{Sinh}[c+d\,\#1]-\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}\left[d\,(x-\#1)\right]+\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}\left[d\,(x-\#1)\right]\right)\,\&\right]+a\,d^2\,\text{RootSum}\left[a+b\,\#1^3\,\&,\ \frac{1}{\#1}\left(\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}\left[d\,(x-\#1)\right]+\text{CoshIntegral}\left[d\,(x-\#1)\right]\text{Sinh}[c+d\,\#1]+\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}\left[d\,(x-\#1)\right]+\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}\left[d\,(x-\#1)\right]\right)\,\&\right]+6\,b\,\left(\text{Cosh}[c+d\,x]-d\,x\,\text{Sinh}[c+d\,x]\right)\right)$$

## Problem 95: Result is not expressed in closed-form.

$$\int\frac{x^3\,\text{Cosh}[c+d\,x]}{a+b\,x^3}\,dx$$

Optimal (type 4, 358 leaves, 14 steps):

$$\frac{(-1)^{1/3}a^{1/3}\,\text{Cosh}\left[c+\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,b^{4/3}}-\frac{(-1)^{2/3}a^{1/3}\,\text{Cosh}\left[c-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]\text{CoshIntegral}\left[-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,b^{4/3}}-\frac{a^{1/3}\,\text{Cosh}\left[c-\frac{a^{1/3}d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,b^{4/3}}+\frac{\text{Sinh}[c+d\,x]}{b\,d}-\frac{(-1)^{1/3}a^{1/3}\,\text{Sinh}\left[c+\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,b^{4/3}}-\frac{a^{1/3}\,\text{Sinh}\left[c-\frac{a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,b^{4/3}}-\frac{(-1)^{2/3}a^{1/3}\,\text{Sinh}\left[c-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,b^{4/3}}$$

Result (type 7, 198 leaves):

$$-\frac{1}{6 b^2 d}\Big(a\, d\, \text{RootSum}\Big[a+b\, \#1^3\, \&,\ \frac{1}{\#1^2}\big(\text{Cosh}[c+d\, \#1]\ \text{CoshIntegral}\big[d\,(x-\#1)\big]-\text{CoshIntegral}\big[d\,(x-\#1)\big]\ \text{Sinh}[c+d\, \#1]-\text{Cosh}[c+d\, \#1]\ \text{SinhIntegral}\big[d\,(x-\#1)\big]+\text{Sinh}[c+d\, \#1]\ \text{SinhIntegral}\big[d\,(x-\#1)\big]\big)\ \&\Big]+a\, d\, \text{RootSum}\Big[a+b\, \#1^3\, \&,\ \frac{1}{\#1^2}\big(\text{Cosh}[c+d\, \#1]\ \text{CoshIntegral}\big[d\,(x-\#1)\big]+\text{CoshIntegral}\big[d\,(x-\#1)\big]\ \text{Sinh}[c+d\, \#1]+\text{Cosh}[c+d\, \#1]\ \text{SinhIntegral}\big[d\,(x-\#1)\big]+\text{Sinh}[c+d\, \#1]\ \text{SinhIntegral}\big[d\,(x-\#1)\big]\big)\ \&\Big]-6\, b\, \text{Sinh}[c+d\, x]\Big)$$

## Problem 96: Result is not expressed in closed-form.

$$\int \frac{x^2\, \text{Cosh}[c+d\, x]}{a+b\, x^3}\, dx$$

Optimal (type 4, 283 leaves, 11 steps):

$$\frac{\text{Cosh}\Big[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\Big]\ \text{CoshIntegral}\Big[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d\, x\Big]}{3 b}+\frac{\text{Cosh}\Big[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\Big]\ \text{CoshIntegral}\Big[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}-d\, x\Big]}{3 b}+\frac{\text{Cosh}\Big[c-\frac{a^{1/3} d}{b^{1/3}}\Big]\ \text{CoshIntegral}\Big[\frac{a^{1/3} d}{b^{1/3}}+d\, x\Big]}{3 b}-\frac{\text{Sinh}\Big[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\Big]\ \text{SinhIntegral}\Big[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d\, x\Big]}{3 b}+\frac{\text{Sinh}\Big[c-\frac{a^{1/3} d}{b^{1/3}}\Big]\ \text{SinhIntegral}\Big[\frac{a^{1/3} d}{b^{1/3}}+d\, x\Big]}{3 b}+\frac{\text{Sinh}\Big[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\Big]\ \text{SinhIntegral}\Big[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}+d\, x\Big]}{3 b}$$

Result (type 7, 170 leaves):

$$\frac{1}{6 b}\Big(\text{RootSum}\big[a+b\, \#1^3\, \&,\ \text{Cosh}[c+d\, \#1]\ \text{CoshIntegral}\big[d\,(x-\#1)\big]-\text{CoshIntegral}\big[d\,(x-\#1)\big]\ \text{Sinh}[c+d\, \#1]-\text{Cosh}[c+d\, \#1]\ \text{SinhIntegral}\big[d\,(x-\#1)\big]+\text{Sinh}[c+d\, \#1]\ \text{SinhIntegral}\big[d\,(x-\#1)\big]\ \&\big]+\text{RootSum}\big[a+b\, \#1^3\, \&,\ \text{Cosh}[c+d\, \#1]\ \text{CoshIntegral}\big[d\,(x-\#1)\big]+\text{CoshIntegral}\big[d\,(x-\#1)\big]\ \text{Sinh}[c+d\, \#1]+\text{Cosh}[c+d\, \#1]\ \text{SinhIntegral}\big[d\,(x-\#1)\big]+\text{Sinh}[c+d\, \#1]\ \text{SinhIntegral}\big[d\,(x-\#1)\big]\ \&\big]\Big)$$

## Problem 97: Result is not expressed in closed-form.

$$\int \frac{x\, \text{Cosh}[c+d\, x]}{a+b\, x^3}\, dx$$

Optimal (type 4, 345 leaves, 11 steps):

$$-\frac{(-1)^{2/3}\operatorname{Cosh}\left[c+\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\operatorname{CoshIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,a^{1/3}\,b^{2/3}}+\frac{(-1)^{1/3}\operatorname{Cosh}\left[c-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]\operatorname{CoshIntegral}\left[-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,a^{1/3}\,b^{2/3}}-\frac{\operatorname{Cosh}\left[c-\frac{a^{1/3}d}{b^{1/3}}\right]\operatorname{CoshIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,a^{1/3}\,b^{2/3}}+\frac{(-1)^{2/3}\operatorname{Sinh}\left[c+\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\operatorname{SinhIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,a^{1/3}\,b^{2/3}}-\frac{\operatorname{Sinh}\left[c-\frac{a^{1/3}d}{b^{1/3}}\right]\operatorname{SinhIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,a^{1/3}\,b^{2/3}}+\frac{(-1)^{1/3}\operatorname{Sinh}\left[c-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]\operatorname{SinhIntegral}\left[\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,a^{1/3}\,b^{2/3}}$$

Result (type 7, 180 leaves):

$$\frac{1}{6\,b}\left(\operatorname{RootSum}\left[a+b\,\#1^3\,\&,\ \frac{1}{\#1}\left(\operatorname{Cosh}[c+d\,\#1]\operatorname{CoshIntegral}\left[d\,(x-\#1)\right]-\operatorname{CoshIntegral}\left[d\,(x-\#1)\right]\operatorname{Sinh}[c+d\,\#1]-\operatorname{Cosh}[c+d\,\#1]\operatorname{SinhIntegral}\left[d\,(x-\#1)\right]+\operatorname{Sinh}[c+d\,\#1]\operatorname{SinhIntegral}\left[d\,(x-\#1)\right]\right)\&\right]+\operatorname{RootSum}\left[a+b\,\#1^3\,\&,\ \frac{1}{\#1}\left(\operatorname{Cosh}[c+d\,\#1]\operatorname{CoshIntegral}\left[d\,(x-\#1)\right]+\operatorname{CoshIntegral}\left[d\,(x-\#1)\right]\operatorname{Sinh}[c+d\,\#1]+\operatorname{Cosh}[c+d\,\#1]\operatorname{SinhIntegral}\left[d\,(x-\#1)\right]+\operatorname{Sinh}[c+d\,\#1]\operatorname{SinhIntegral}\left[d\,(x-\#1)\right]\right)\&\right]\right)$$

## Problem 98: Result is not expressed in closed-form.

$$\int\frac{\operatorname{Cosh}[c+d\,x]}{a+b\,x^3}\,dx$$

Optimal (type 4, 345 leaves, 11 steps):

$$-\frac{(-1)^{1/3}\operatorname{Cosh}\left[c+\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\operatorname{CoshIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,a^{2/3}\,b^{1/3}}+\frac{(-1)^{2/3}\operatorname{Cosh}\left[c-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]\operatorname{CoshIntegral}\left[-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,a^{2/3}\,b^{1/3}}+\frac{\operatorname{Cosh}\left[c-\frac{a^{1/3}d}{b^{1/3}}\right]\operatorname{CoshIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,a^{2/3}\,b^{1/3}}+\frac{(-1)^{1/3}\operatorname{Sinh}\left[c+\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\operatorname{SinhIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,a^{2/3}\,b^{1/3}}+\frac{\operatorname{Sinh}\left[c-\frac{a^{1/3}d}{b^{1/3}}\right]\operatorname{SinhIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,a^{2/3}\,b^{1/3}}+\frac{(-1)^{2/3}\operatorname{Sinh}\left[c-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]\operatorname{SinhIntegral}\left[\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,a^{2/3}\,b^{1/3}}$$

Result (type 7, 180 leaves):

$$\frac{1}{6\,b}\left(\text{RootSum}\left[a+b\,\#1^3\,\&,\ \frac{1}{\#1^2}\left(\text{Cosh}[c+d\,\#1]\ \text{CoshIntegral}\left[d\left(x-\#1\right)\right]-\text{CoshIntegral}\left[d\left(x-\#1\right)\right]\ \text{Sinh}[c+d\,\#1]-\text{Cosh}[c+d\,\#1]\ \text{SinhIntegral}\left[d\left(x-\#1\right)\right]+\text{Sinh}[c+d\,\#1]\ \text{SinhIntegral}\left[d\left(x-\#1\right)\right]\right)\&\right]+\text{RootSum}\left[a+b\,\#1^3\,\&,\ \frac{1}{\#1^2}\left(\text{Cosh}[c+d\,\#1]\ \text{CoshIntegral}\left[d\left(x-\#1\right)\right]+\text{CoshIntegral}\left[d\left(x-\#1\right)\right]\ \text{Sinh}[c+d\,\#1]+\text{Cosh}[c+d\,\#1]\ \text{SinhIntegral}\left[d\left(x-\#1\right)\right]+\text{Sinh}[c+d\,\#1]\ \text{SinhIntegral}\left[d\left(x-\#1\right)\right]\right)\&\right]\right)$$

## Problem 99: Result is not expressed in closed-form.

$$\int \frac{\text{Cosh}[c+d\,x]}{x\left(a+b\,x^3\right)}\,dx$$

Optimal (type 4, 303 leaves, 16 steps):

$$\frac{\text{Cosh}[c]\ \text{CoshIntegral}[d\,x]}{a}-\frac{\text{Cosh}\left[c+\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\ \text{CoshIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,a}-\frac{\text{Cosh}\left[c-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]\ \text{CoshIntegral}\left[-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,a}-\frac{\text{Cosh}\left[c-\frac{a^{1/3}d}{b^{1/3}}\right]\ \text{CoshIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,a}+\frac{\text{Sinh}[c]\ \text{SinhIntegral}[d\,x]}{a}+\frac{\text{Sinh}\left[c+\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\ \text{SinhIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,a}-\frac{\text{Sinh}\left[c-\frac{a^{1/3}d}{b^{1/3}}\right]\ \text{SinhIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,a}-\frac{\text{Sinh}\left[c-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]\ \text{SinhIntegral}\left[\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,a}$$

Result (type 7, 186 leaves):

$$-\frac{1}{6\,a}\left(-6\,\text{Cosh}[c]\ \text{CoshIntegral}[d\,x]+\text{RootSum}\left[a+b\,\#1^3\,\&,\ \text{Cosh}[c+d\,\#1]\ \text{CoshIntegral}\left[d\left(x-\#1\right)\right]-\text{CoshIntegral}\left[d\left(x-\#1\right)\right]\ \text{Sinh}[c+d\,\#1]-\text{Cosh}[c+d\,\#1]\ \text{SinhIntegral}\left[d\left(x-\#1\right)\right]+\text{Sinh}[c+d\,\#1]\ \text{SinhIntegral}\left[d\left(x-\#1\right)\right]\&\right]+\text{RootSum}\left[a+b\,\#1^3\,\&,\ \text{Cosh}[c+d\,\#1]\ \text{CoshIntegral}\left[d\left(x-\#1\right)\right]+\text{CoshIntegral}\left[d\left(x-\#1\right)\right]\ \text{Sinh}[c+d\,\#1]+\text{Cosh}[c+d\,\#1]\ \text{SinhIntegral}\left[d\left(x-\#1\right)\right]+\text{Sinh}[c+d\,\#1]\ \text{SinhIntegral}\left[d\left(x-\#1\right)\right]\&\right]-6\,\text{Sinh}[c]\ \text{SinhIntegral}[d\,x]\right)$$

## Problem 100: Result is not expressed in closed-form.

$$\int \frac{\text{Cosh}[c+d\,x]}{x^2\left(a+b\,x^3\right)}\,dx$$

Optimal (type 4, 381 leaves, 17 steps):

$$-\frac{\text{Cosh}[c+d\,x]}{a\,x}+\frac{(-1)^{2/3}\,b^{1/3}\,\text{Cosh}\left[c+\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,a^{4/3}}-$$
$$\frac{(-1)^{1/3}\,b^{1/3}\,\text{Cosh}\left[c-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]\text{CoshIntegral}\left[-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,a^{4/3}}+\frac{b^{1/3}\,\text{Cosh}\left[c-\frac{a^{1/3}d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,a^{4/3}}+$$
$$\frac{d\,\text{CoshIntegral}[d\,x]\,\text{Sinh}[c]}{a}+\frac{d\,\text{Cosh}[c]\,\text{SinhIntegral}[d\,x]}{a}-\frac{(-1)^{2/3}\,b^{1/3}\,\text{Sinh}\left[c+\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,a^{4/3}}+$$
$$\frac{b^{1/3}\,\text{Sinh}\left[c-\frac{a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,a^{4/3}}-\frac{(-1)^{1/3}\,b^{1/3}\,\text{Sinh}\left[c-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,a^{4/3}}$$

Result (type 7, 215 leaves) :

$$-\frac{1}{6\,a\,x}\Big(6\,\text{Cosh}[c+d\,x]+x\,\text{RootSum}\Big[a+b\,\#1^3\,\&,\ \frac{1}{\#1}\big(\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]-\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]-\text{Cosh}[c+d\,\#1]$$
$$\text{SinhIntegral}[d\,(x-\#1)]+\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\big)\,\&\Big]+$$
$$x\,\text{RootSum}\Big[a+b\,\#1^3\,\&,\ \frac{1}{\#1}\big(\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]+\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]+$$
$$\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]+\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\big)\,\&\Big]-$$
$$6\,d\,x\,\text{CoshIntegral}[d\,x]\,\text{Sinh}[c]-6\,d\,x\,\text{Cosh}[c]\,\text{SinhIntegral}[d\,x]\Big)$$

## Problem 101: Result is not expressed in closed-form.

$$\int\frac{\text{Cosh}[c+d\,x]}{x^3\,(a+b\,x^3)}\,dx$$

Optimal (type 4, 410 leaves, 18 steps) :

$$-\frac{\text{Cosh}[c+d x]}{2 a x^2}+\frac{d^2 \text{Cosh}[c] \text{CoshIntegral}[d x]}{2 a}+\frac{(-1)^{1/3} b^{2/3} \text{Cosh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{3 a^{5/3}}-$$
$$\frac{(-1)^{2/3} b^{2/3} \text{Cosh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}-d x\right]}{3 a^{5/3}}-\frac{b^{2/3} \text{Cosh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \text{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{3 a^{5/3}}-$$
$$\frac{d \text{Sinh}[c+d x]}{2 a x}+\frac{d^2 \text{Sinh}[c] \text{SinhIntegral}[d x]}{2 a}-\frac{(-1)^{1/3} b^{2/3} \text{Sinh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{3 a^{5/3}}-$$
$$\frac{b^{2/3} \text{Sinh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{3 a^{5/3}}-\frac{(-1)^{2/3} b^{2/3} \text{Sinh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}+d x\right]}{3 a^{5/3}}$$

Result (type 7, 237 leaves) :

$$-\frac{1}{6 a x^2}\Big(3 \text{Cosh}[c+d x]-3 d^2 x^2 \text{Cosh}[c] \text{CoshIntegral}[d x]+$$
$$x^2 \text{RootSum}\Big[a+b \#1^3 \&,\frac{1}{\#1^2}\Big(\text{Cosh}[c+d \#1] \text{CoshIntegral}[d (x-\#1)]-\text{CoshIntegral}[d (x-\#1)] \text{Sinh}[c+d \#1]-$$
$$\text{Cosh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)]+\text{Sinh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)]\Big) \&\Big]+$$
$$x^2 \text{RootSum}\Big[a+b \#1^3 \&,\frac{1}{\#1^2}\Big(\text{Cosh}[c+d \#1] \text{CoshIntegral}[d (x-\#1)]+\text{CoshIntegral}[d (x-\#1)] \text{Sinh}[c+d \#1]+\text{Cosh}[c+d \#1]$$
$$\text{SinhIntegral}[d (x-\#1)]+\text{Sinh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)]\Big) \&\Big]+3 d x \text{Sinh}[c+d x]-3 d^2 x^2 \text{Sinh}[c] \text{SinhIntegral}[d x]\Big)$$

## Problem 102: Result is not expressed in closed-form.

$$\int \frac{x^3 \text{Cosh}[c+d x]}{\left(a+b x^3\right)^2} \, dx$$

Optimal (type 4, 718 leaves, 23 steps) :

$$-\frac{x\,\text{Cosh}[c+d\,x]}{3\,b\,(a+b\,x^3)}-\frac{(-1)^{1/3}\,\text{Cosh}\left[c+\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}-d\,x\right]}{9\,a^{2/3}\,b^{4/3}}+\frac{(-1)^{2/3}\,\text{Cosh}\left[c-\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}\right]\text{CoshIntegral}\left[-\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}-d\,x\right]}{9\,a^{2/3}\,b^{4/3}}+$$

$$\frac{\text{Cosh}\left[c-\frac{a^{1/3}\,d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{a^{1/3}\,d}{b^{1/3}}+d\,x\right]}{9\,a^{2/3}\,b^{4/3}}-\frac{d\,\text{CoshIntegral}\left[\frac{a^{1/3}\,d}{b^{1/3}}+d\,x\right]\text{Sinh}\left[c-\frac{a^{1/3}\,d}{b^{1/3}}\right]}{9\,a^{1/3}\,b^{5/3}}-$$

$$\frac{(-1)^{2/3}\,d\,\text{CoshIntegral}\left[\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}-d\,x\right]\text{Sinh}\left[c+\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}\right]}{9\,a^{1/3}\,b^{5/3}}+\frac{(-1)^{1/3}\,d\,\text{CoshIntegral}\left[-\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}-d\,x\right]\text{Sinh}\left[c-\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}\right]}{9\,a^{1/3}\,b^{5/3}}+$$

$$\frac{(-1)^{2/3}\,d\,\text{Cosh}\left[c+\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}-d\,x\right]}{9\,a^{1/3}\,b^{5/3}}+\frac{(-1)^{1/3}\,\text{Sinh}\left[c+\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}-d\,x\right]}{9\,a^{2/3}\,b^{4/3}}-$$

$$\frac{d\,\text{Cosh}\left[c-\frac{a^{1/3}\,d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{a^{1/3}\,d}{b^{1/3}}+d\,x\right]}{9\,a^{1/3}\,b^{5/3}}+\frac{\text{Sinh}\left[c-\frac{a^{1/3}\,d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{a^{1/3}\,d}{b^{1/3}}+d\,x\right]}{9\,a^{2/3}\,b^{4/3}}+$$

$$\frac{(-1)^{1/3}\,d\,\text{Cosh}\left[c-\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}+d\,x\right]}{9\,a^{1/3}\,b^{5/3}}+\frac{(-1)^{2/3}\,\text{Sinh}\left[c-\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}+d\,x\right]}{9\,a^{2/3}\,b^{4/3}}$$

Result (type 7, 363 leaves):

$$\frac{1}{18\,b^2}\left(-\frac{6\,b\,x\,\text{Cosh}[c+d\,x]}{a+b\,x^3}-\text{RootSum}\Big[a+b\,\sharp1^3\,\&,\right.$$
$$\frac{1}{\sharp1^2}\Big(-\text{Cosh}[c+d\,\sharp1]\,\text{CoshIntegral}[d\,(x-\sharp1)]+\text{CoshIntegral}[d\,(x-\sharp1)]\,\text{Sinh}[c+d\,\sharp1]+\text{Cosh}[c+d\,\sharp1]\,\text{SinhIntegral}[d\,(x-\sharp1)]-$$
$$\text{Sinh}[c+d\,\sharp1]\,\text{SinhIntegral}[d\,(x-\sharp1)]+d\,\text{Cosh}[c+d\,\sharp1]\,\text{CoshIntegral}[d\,(x-\sharp1)]\,\sharp1-d\,\text{CoshIntegral}[d\,(x-\sharp1)]\,\text{Sinh}[c+d\,\sharp1]\,\sharp1-$$
$$d\,\text{Cosh}[c+d\,\sharp1]\,\text{SinhIntegral}[d\,(x-\sharp1)]\,\sharp1+d\,\text{Sinh}[c+d\,\sharp1]\,\text{SinhIntegral}[d\,(x-\sharp1)]\,\sharp1\Big)\,\&\Big]+\text{RootSum}\Big[a+b\,\sharp1^3\,\&,$$
$$\frac{1}{\sharp1^2}\Big(\text{Cosh}[c+d\,\sharp1]\,\text{CoshIntegral}[d\,(x-\sharp1)]+\text{CoshIntegral}[d\,(x-\sharp1)]\,\text{Sinh}[c+d\,\sharp1]+\text{Cosh}[c+d\,\sharp1]\,\text{SinhIntegral}[d\,(x-\sharp1)]+$$
$$\text{Sinh}[c+d\,\sharp1]\,\text{SinhIntegral}[d\,(x-\sharp1)]+d\,\text{Cosh}[c+d\,\sharp1]\,\text{CoshIntegral}[d\,(x-\sharp1)]\,\sharp1+d\,\text{CoshIntegral}[d\,(x-\sharp1)]\,\text{Sinh}[c+d\,\sharp1]\,\sharp1+$$
$$\left.d\,\text{Cosh}[c+d\,\sharp1]\,\text{SinhIntegral}[d\,(x-\sharp1)]\,\sharp1+d\,\text{Sinh}[c+d\,\sharp1]\,\text{SinhIntegral}[d\,(x-\sharp1)]\,\sharp1\Big)\,\&\Big]\right)$$

## Problem 103: Result is not expressed in closed-form.

$$\int\frac{x^2\,\text{Cosh}[c+d\,x]}{(a+b\,x^3)^2}\,dx$$

Optimal (type 4, 373 leaves, 12 steps):

$$-\frac{\text{Cosh}[c+d x]}{3 b \left(a+b x^3\right)}+\frac{d\, \text{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right] \text{Sinh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right]}{9 a^{2/3} b^{4/3}}-\frac{(-1)^{1/3} d\, \text{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right] \text{Sinh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{9 a^{2/3} b^{4/3}}+$$
$$\frac{(-1)^{2/3} d\, \text{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}-d x\right] \text{Sinh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{9 a^{2/3} b^{4/3}}+\frac{(-1)^{1/3} d\, \text{Cosh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{9 a^{2/3} b^{4/3}}+$$
$$\frac{d\, \text{Cosh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{9 a^{2/3} b^{4/3}}+\frac{(-1)^{2/3} d\, \text{Cosh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}+d x\right]}{9 a^{2/3} b^{4/3}}$$

Result (type 7, 203 leaves) :

$$\frac{1}{18 b^2}\left(-\frac{6 b\, \text{Cosh}[c+d x]}{a+b x^3}-d\, \text{RootSum}\left[a+b \#1^3 \&, \frac{1}{\#1^2}\left(\text{Cosh}[c+d \#1]\, \text{CoshIntegral}[d (x-\#1)]-\right.\right.\right.$$
$$\left.\text{CoshIntegral}[d (x-\#1)]\, \text{Sinh}[c+d \#1]-\text{Cosh}[c+d \#1]\, \text{SinhIntegral}[d (x-\#1)]+\text{Sinh}[c+d \#1]\, \text{SinhIntegral}[d (x-\#1)]\right) \&]+$$
$$d\, \text{RootSum}\left[a+b \#1^3 \&, \frac{1}{\#1^2}\left(\text{Cosh}[c+d \#1]\, \text{CoshIntegral}[d (x-\#1)]+\text{CoshIntegral}[d (x-\#1)]\, \text{Sinh}[c+d \#1]+\right.\right.$$
$$\left.\left.\left.\text{Cosh}[c+d \#1]\, \text{SinhIntegral}[d (x-\#1)]+\text{Sinh}[c+d \#1]\, \text{SinhIntegral}[d (x-\#1)]\right) \&\right]\right)$$

## Problem 104: Result is not expressed in closed-form.

$$\int \frac{x\, \text{Cosh}[c+d x]}{\left(a+b x^3\right)^2}\, dx$$

Optimal (type 4, 695 leaves, 34 steps) :

$$\frac{\text{Cosh}[c+d x]}{3 a b x}-\frac{\text{Cosh}[c+d x]}{3 b x\left(a+b x^3\right)}-\frac{(-1)^{2/3}\,\text{Cosh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{9 a^{4/3} b^{2/3}}+$$
$$\frac{(-1)^{1/3}\,\text{Cosh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]\text{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}-d x\right]}{9 a^{4/3} b^{2/3}}-\frac{\text{Cosh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{9 a^{4/3} b^{2/3}}-$$
$$\frac{d\,\text{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]\text{Sinh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right]}{9 a b}-\frac{d\,\text{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]\text{Sinh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{9 a b}-$$
$$\frac{d\,\text{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}-d x\right]\text{Sinh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{9 a b}+\frac{d\,\text{Cosh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{9 a b}+$$
$$\frac{(-1)^{2/3}\,\text{Sinh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{9 a^{4/3} b^{2/3}}-\frac{d\,\text{Cosh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{9 a b}-$$
$$\frac{\text{Sinh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{9 a^{4/3} b^{2/3}}-\frac{d\,\text{Cosh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}+d x\right]}{9 a b}+$$
$$\frac{(-1)^{1/3}\,\text{Sinh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}+d x\right]}{9 a^{4/3} b^{2/3}}$$

Result (type 7, 387 leaves) :

$$\frac{1}{18 a b\left(a+b x^3\right)}\Bigg(6 b x^2\,\text{Cosh}[c+d x]+\left(a+b x^3\right)\text{RootSum}\Big[a+b\,\#1^3\,\&,$$
$$\frac{1}{\#1}\Big(\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]-\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]-\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]+$$
$$\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]+d\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]\,\#1-d\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]\,\#1-$$
$$d\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1+d\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1\Big)\,\&\Big]-\left(a+b x^3\right)\text{RootSum}\Big[a+b\,\#1^3\,\&,$$
$$\frac{1}{\#1}\Big(-\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]-\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]-\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]-$$
$$\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]+d\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]\,\#1+d\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]\,\#1+$$
$$d\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1+d\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1\Big)\,\&\Big]\Bigg)$$

## Problem 105: Result is not expressed in closed-form.

$$\int\frac{\text{Cosh}[c+d x]}{\left(a+b x^3\right)^2}\,dx$$

Optimal (type 4, 739 leaves, 36 steps) :

$$\frac{\text{Cosh}[c+d x]}{3 a b x^2}-\frac{\text{Cosh}[c+d x]}{3 b x^2 \left(a+b x^3\right)}-\frac{2 (-1)^{1/3} \text{Cosh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{9 a^{5/3} b^{1/3}}+$$

$$\frac{2 (-1)^{2/3} \text{Cosh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}-d x\right]}{9 a^{5/3} b^{1/3}}+\frac{2 \text{Cosh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \text{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{9 a^{5/3} b^{1/3}}+$$

$$\frac{d\, \text{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right] \text{Sinh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right]}{9 a^{4/3} b^{2/3}}+\frac{(-1)^{2/3} d\, \text{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right] \text{Sinh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{9 a^{4/3} b^{2/3}}-$$

$$\frac{(-1)^{1/3} d\, \text{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}-d x\right] \text{Sinh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{9 a^{4/3} b^{2/3}}-\frac{(-1)^{2/3} d\, \text{Cosh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{9 a^{4/3} b^{2/3}}+$$

$$\frac{2 (-1)^{1/3} \text{Sinh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{9 a^{5/3} b^{1/3}}+\frac{d\, \text{Cosh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{9 a^{4/3} b^{2/3}}+$$

$$\frac{2 \text{Sinh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{9 a^{5/3} b^{1/3}}-\frac{(-1)^{1/3} d\, \text{Cosh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}+d x\right]}{9 a^{4/3} b^{2/3}}+$$

$$\frac{2 (-1)^{2/3} \text{Sinh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}+d x\right]}{9 a^{5/3} b^{1/3}}$$

Result (type 7, 387 leaves) :

$$\frac{1}{18 a b \left(a+b x^3\right)}\Bigg(6 b x \text{Cosh}[c+d x]+\left(a+b x^3\right) \text{RootSum}\Big[a+b \#1^3 \&,$$
$$\frac{1}{\#1^2}\Big(2 \text{Cosh}[c+d \#1] \text{CoshIntegral}[d (x-\#1)]-2 \text{CoshIntegral}[d (x-\#1)] \text{Sinh}[c+d \#1]-2 \text{Cosh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)]+$$
$$2 \text{Sinh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)]+d \text{Cosh}[c+d \#1] \text{CoshIntegral}[d (x-\#1)] \#1-d \text{CoshIntegral}[d (x-\#1)] \text{Sinh}[c+d \#1] \#1-$$
$$d \text{Cosh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)] \#1+d \text{Sinh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)] \#1\Big) \&\Big]-\left(a+b x^3\right) \text{RootSum}\Big[a+b \#1^3 \&,$$
$$\frac{1}{\#1^2}\Big(-2 \text{Cosh}[c+d \#1] \text{CoshIntegral}[d (x-\#1)]-2 \text{CoshIntegral}[d (x-\#1)] \text{Sinh}[c+d \#1]-2 \text{Cosh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)]-$$
$$2 \text{Sinh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)]+d \text{Cosh}[c+d \#1] \text{CoshIntegral}[d (x-\#1)] \#1+d \text{CoshIntegral}[d (x-\#1)] \text{Sinh}[c+d \#1] \#1+$$
$$d \text{Cosh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)] \#1+d \text{Sinh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)] \#1\Big) \&\Big]\Bigg)$$

## Problem 106: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cosh}[c+d x]}{x \left(a+b x^3\right)^2} \, dx$$

Optimal (type 4, 697 leaves, 41 steps) :

$$\frac{\text{Cosh}[c+d\,x]}{3\,a\,b\,x^3}-\frac{\text{Cosh}[c+d\,x]}{3\,b\,x^3\,(a+b\,x^3)}+\frac{\text{Cosh}[c]\,\text{CoshIntegral}[d\,x]}{a^2}-\frac{\text{Cosh}\left[c+\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,a^2}-$$

$$\frac{\text{Cosh}\left[c-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]\text{CoshIntegral}\left[-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,a^2}-\frac{\text{Cosh}\left[c-\frac{a^{1/3}d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,a^2}-\frac{d\,\text{CoshIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]\text{Sinh}\left[c-\frac{a^{1/3}d}{b^{1/3}}\right]}{9\,a^{5/3}\,b^{1/3}}+$$

$$\frac{(-1)^{1/3}d\,\text{CoshIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]\text{Sinh}\left[c+\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]}{9\,a^{5/3}\,b^{1/3}}-\frac{(-1)^{2/3}d\,\text{CoshIntegral}\left[-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}-d\,x\right]\text{Sinh}\left[c-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]}{9\,a^{5/3}\,b^{1/3}}+$$

$$\frac{\text{Sinh}[c]\,\text{SinhIntegral}[d\,x]}{a^2}-\frac{(-1)^{1/3}d\,\text{Cosh}\left[c+\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{9\,a^{5/3}\,b^{1/3}}+$$

$$\frac{\text{Sinh}\left[c+\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]}{3\,a^2}-\frac{d\,\text{Cosh}\left[c-\frac{a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]}{9\,a^{5/3}\,b^{1/3}}-\frac{\text{Sinh}\left[c-\frac{a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,a^2}-$$

$$\frac{(-1)^{2/3}d\,\text{Cosh}\left[c-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}+d\,x\right]}{9\,a^{5/3}\,b^{1/3}}-\frac{\text{Sinh}\left[c-\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}+d\,x\right]}{3\,a^2}$$

Result (type 4, 5530 leaves) :

$$\text{Sinh}[c]\left(\frac{\text{SinhIntegral}[d\,x]}{a^2}-\right.$$

$$\left(\left(2\,b^{1/3}-3\,(-1)^{1/3}b^{1/3}+3\,(-1)^{2/3}b^{1/3}\right)\left(-\text{CoshIntegral}\left[d\left(\frac{a^{1/3}}{b^{1/3}}+x\right)\right]\text{Sinh}\left[\frac{a^{1/3}d}{b^{1/3}}\right]+\text{Cosh}\left[\frac{a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[d\left(\frac{a^{1/3}}{b^{1/3}}+x\right)\right]\right)\right)\Big/$$

$$\left(\left(-1+(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^2\,b^{1/3}\right)+\left(\left(21-22\,(-1)^{1/3}+21\,(-1)^{2/3}\right)b^{1/3}\left(-\frac{\text{Sinh}[d\,x]}{b^{1/3}\left(-(-1)^{1/3}a^{1/3}+b^{1/3}x\right)}+\frac{1}{b^{2/3}}\right.\right.$$

$$\left.\left.d\left(\text{Cosh}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\text{CoshIntegral}\left[-\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}+d\,x\right]-\text{Sinh}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]\right)\right)\right)\Big/$$

$$\left(3\left(-1+(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^{5/3}\right)+\left(\left(22-21\,(-1)^{1/3}+21\,(-1)^{2/3}\right)b^{1/3}\left(-\frac{\text{Sinh}[d\,x]}{b^{1/3}\left(a^{1/3}+b^{1/3}x\right)}+\right.\right.$$

$$\left.\left.\frac{d\left(\text{Cosh}\left[\frac{a^{1/3}d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]-\text{Sinh}\left[\frac{a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]\right)}{b^{2/3}}\right)\right)\Big/\left(3\left(-1+(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^{5/3}\right)+$$

$$\frac{1}{3\,(1+(-1)^{1/3})^2\,a^{5/3}}\,(22\,b^{1/3}-21\,(-1)^{1/3}\,b^{1/3}+21\,(-1)^{2/3}\,b^{1/3})\,\Bigg(-\frac{\text{Sinh}[d\,x]}{b^{1/3}\,((-1)^{2/3}\,a^{1/3}+b^{1/3}\,x)}+\frac{1}{b^{2/3}}$$

$$d\,\Bigg(\text{Cosh}\Big[\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}\Big]\,\text{CoshIntegral}\Big[\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}+d\,x\Big]-\text{Sinh}\Big[\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}\Big]\,\text{SinhIntegral}\Big[\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}+d\,x\Big]\Bigg)\Bigg)+$$

$$\Bigg(i\,(2\,b^{1/3}-3\,(-1)^{1/3}\,b^{1/3}+3\,(-1)^{2/3}\,b^{1/3})\,\Bigg(\text{CosIntegral}\Big[-\frac{(-1)^{1/6}\,a^{1/3}\,d}{b^{1/3}}+i\,d\,x\Big]\,\text{Sin}\Big[\frac{(-1)^{1/6}\,a^{1/3}\,d}{b^{1/3}}\Big]-$$

$$\text{Cos}\Big[\frac{(-1)^{1/6}\,a^{1/3}\,d}{b^{1/3}}\Big]\,\text{SinIntegral}\Big[\frac{(-1)^{1/6}\,a^{1/3}\,d}{b^{1/3}}-i\,d\,x\Big]\Bigg)\Bigg)\Big/\,\Big((-1+(-1)^{1/3})\,(1+(-1)^{1/3})^2\,a^2\,b^{1/3}\Big)+$$

$$\frac{1}{(1+(-1)^{1/3})^2\,a^2\,b^{1/3}}\,i\,(3\,b^{1/3}-2\,(-1)^{1/3}\,b^{1/3}+3\,(-1)^{2/3}\,b^{1/3})\,\Bigg(\text{CosIntegral}\Big[-\frac{(-1)^{5/6}\,a^{1/3}\,d}{b^{1/3}}+i\,d\,x\Big]\,\text{Sin}\Big[\frac{(-1)^{5/6}\,a^{1/3}\,d}{b^{1/3}}\Big]-$$

$$\text{Cos}\Big[\frac{(-1)^{5/6}\,a^{1/3}\,d}{b^{1/3}}\Big]\,\text{SinIntegral}\Big[\frac{(-1)^{5/6}\,a^{1/3}\,d}{b^{1/3}}-i\,d\,x\Big]\Bigg)\Bigg)+\text{Cosh}[c]\,\Bigg(\frac{\text{CoshIntegral}[d\,x]}{a^2}+$$

$$\Bigg((22-21\,(-1)^{1/3}+21\,(-1)^{2/3})\,\Bigg(-\frac{b^{1/3}\,\text{Cosh}[d\,x]}{a^{1/3}+b^{1/3}\,x}-d\,\text{CoshIntegral}\Big[d\,\Big(\frac{a^{1/3}}{b^{1/3}}+x\Big)\Big]\,\text{Sinh}\Big[\frac{a^{1/3}\,d}{b^{1/3}}\Big]+d\,\text{Cosh}\Big[\frac{a^{1/3}\,d}{b^{1/3}}\Big]\,\text{SinhIntegral}\Big[d\,\Big(\frac{a^{1/3}}{b^{1/3}}+x\Big)\Big]\Bigg)\Bigg)\Big/$$

$$\Big(3\,(-1+(-1)^{1/3})\,(1+(-1)^{1/3})^2\,a^{5/3}\,b^{1/3}\Big)+$$

$$\Bigg((21-22\,(-1)^{1/3}+21\,(-1)^{2/3})\,b^{1/3}\,\Bigg(\frac{\text{Cosh}[d\,x]}{b^{1/3}\,((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x)}+\frac{1}{b^{2/3}}d\,\Bigg(\text{CoshIntegral}\Big[d\,\Big(-\frac{(-1)^{1/3}\,a^{1/3}}{b^{1/3}}+x\Big)\Big]\,\text{Sinh}\Big[\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}\Big]-$$

$$\text{Cosh}\Big[\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}\Big]\,\text{SinhIntegral}\Big[\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}-d\,x\Big]\Bigg)\Bigg)\Bigg)\Big/\,\Big(3\,(-1+(-1)^{1/3})\,(1+(-1)^{1/3})^2\,a^{5/3}\Big)-$$

$$\Bigg((2\,b^{1/3}-3\,(-1)^{1/3}\,b^{1/3}+3\,(-1)^{2/3}\,b^{1/3})\,\Bigg(\text{Cosh}\Big[\frac{a^{1/3}\,d}{b^{1/3}}\Big]\,\text{CoshIntegral}\Big[\frac{a^{1/3}\,d}{b^{1/3}}+d\,x\Big]-\text{Sinh}\Big[\frac{a^{1/3}\,d}{b^{1/3}}\Big]\,\text{SinhIntegral}\Big[\frac{a^{1/3}\,d}{b^{1/3}}+d\,x\Big]\Bigg)\Bigg)\Big/$$

$$\Big((-1+(-1)^{1/3})\,(1+(-1)^{1/3})^2\,a^2\,b^{1/3}\Big)+\frac{1}{3\,(1+(-1)^{1/3})^2\,a^{5/3}}$$

$$(22\,b^{1/3}-21\,(-1)^{1/3}\,b^{1/3}+21\,(-1)^{2/3}\,b^{1/3})\,\Bigg(-\frac{\text{Cosh}[d\,x]}{b^{1/3}\,((-1)^{2/3}\,a^{1/3}+b^{1/3}\,x)}-\frac{1}{b^{2/3}}$$

$$d\,\Bigg(\text{CoshIntegral}\Big[\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}+d\,x\Big]\,\text{Sinh}\Big[\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}\Big]-\text{Cosh}\Big[\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}\Big]\,\text{SinhIntegral}\Big[\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}+d\,x\Big]\Bigg)\Bigg)-$$

$$\Bigg(\Big(2\,b^{1/3}-3\,(-1)^{1/3}\,b^{1/3}+3\,(-1)^{2/3}\,b^{1/3}\Big)\,\Bigg(\mathrm{Cos}\Big[\frac{(-1)^{1/6}\,a^{1/3}\,d}{b^{1/3}}\Big]\,\mathrm{CosIntegral}\Big[-\frac{(-1)^{1/6}\,a^{1/3}\,d}{b^{1/3}}+\mathbb{i}\,d\,x\Big]+$$

$$\mathrm{Sin}\Big[\frac{(-1)^{1/6}\,a^{1/3}\,d}{b^{1/3}}\Big]\,\mathrm{SinIntegral}\Big[\frac{(-1)^{1/6}\,a^{1/3}\,d}{b^{1/3}}-\mathbb{i}\,d\,x\Big]\Bigg)\Bigg)\Bigg/$$

$$\Bigg(\Big(-1+(-1)^{1/3}\Big)\,\Big(1+(-1)^{1/3}\Big)^2\,a^2\,b^{1/3}\Bigg)-\frac{1}{\Big(1+(-1)^{1/3}\Big)^2\,a^2\,b^{1/3}}\Big(3\,b^{1/3}-2\,(-1)^{1/3}\,b^{1/3}+3\,(-1)^{2/3}\,b^{1/3}\Big)$$

$$\Bigg(\mathrm{Cos}\Big[\frac{(-1)^{5/6}\,a^{1/3}\,d}{b^{1/3}}\Big]\,\mathrm{CosIntegral}\Big[-\frac{(-1)^{5/6}\,a^{1/3}\,d}{b^{1/3}}+\mathbb{i}\,d\,x\Big]+\mathrm{Sin}\Big[\frac{(-1)^{5/6}\,a^{1/3}\,d}{b^{1/3}}\Big]\,\mathrm{SinIntegral}\Big[\frac{(-1)^{5/6}\,a^{1/3}\,d}{b^{1/3}}-\mathbb{i}\,d\,x\Big]\Bigg)\Bigg)+$$

$$\frac{1}{2}\Bigg(-\mathrm{Cosh}[c]\,\Bigg(\frac{\mathrm{SinhIntegral}[d\,x]}{a^2}-\Bigg(\Big(2\,b^{1/3}-3\,(-1)^{1/3}\,b^{1/3}+3\,(-1)^{2/3}\,b^{1/3}\Big)$$

$$\Bigg(-\mathrm{CoshIntegral}\Big[d\,\Big(\frac{a^{1/3}}{b^{1/3}}+x\Big)\Big]\,\mathrm{Sinh}\Big[\frac{a^{1/3}\,d}{b^{1/3}}\Big]+\mathrm{Cosh}\Big[\frac{a^{1/3}\,d}{b^{1/3}}\Big]\,\mathrm{SinhIntegral}\Big[d\,\Big(\frac{a^{1/3}}{b^{1/3}}+x\Big)\Big]\Bigg)\Bigg)\Bigg/$$

$$\Bigg(\Big(-1+(-1)^{1/3}\Big)\,\Big(1+(-1)^{1/3}\Big)^2\,a^2\,b^{1/3}\Bigg)+\Bigg(\Big(21-22\,(-1)^{1/3}+21\,(-1)^{2/3}\Big)\,b^{1/3}\,\Bigg(-\frac{\mathrm{Sinh}[d\,x]}{b^{1/3}\,\Big(-(-1)^{1/3}\,a^{1/3}+b^{1/3}\,x\Big)}+\frac{1}{b^{2/3}}$$

$$d\,\Bigg(\mathrm{Cosh}\Big[\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}\Big]\,\mathrm{CoshIntegral}\Big[-\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}+d\,x\Big]-\mathrm{Sinh}\Big[\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}\Big]\,\mathrm{SinhIntegral}\Big[\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}-d\,x\Big]\Bigg)\Bigg)\Bigg)\Bigg/$$

$$\Bigg(3\,\Big(-1+(-1)^{1/3}\Big)\,\Big(1+(-1)^{1/3}\Big)^2\,a^{5/3}\Bigg)+\Bigg(\Big(22-21\,(-1)^{1/3}+21\,(-1)^{2/3}\Big)\,b^{1/3}\,\Bigg(-\frac{\mathrm{Sinh}[d\,x]}{b^{1/3}\,\Big(a^{1/3}+b^{1/3}\,x\Big)}+$$

$$\frac{d\,\Big(\mathrm{Cosh}\Big[\frac{a^{1/3}\,d}{b^{1/3}}\Big]\,\mathrm{CoshIntegral}\Big[\frac{a^{1/3}\,d}{b^{1/3}}+d\,x\Big]-\mathrm{Sinh}\Big[\frac{a^{1/3}\,d}{b^{1/3}}\Big]\,\mathrm{SinhIntegral}\Big[\frac{a^{1/3}\,d}{b^{1/3}}+d\,x\Big]\Big)}{b^{2/3}}\Bigg)\Bigg)\Bigg/\Bigg(3\,\Big(-1+(-1)^{1/3}\Big)\,\Big(1+(-1)^{1/3}\Big)^2\,a^{5/3}\Bigg)+$$

$$\frac{1}{3\,\Big(1+(-1)^{1/3}\Big)^2\,a^{5/3}}\Big(22\,b^{1/3}-21\,(-1)^{1/3}\,b^{1/3}+21\,(-1)^{2/3}\,b^{1/3}\Big)\,\Bigg(-\frac{\mathrm{Sinh}[d\,x]}{b^{1/3}\,\Big((-1)^{2/3}\,a^{1/3}+b^{1/3}\,x\Big)}+\frac{1}{b^{2/3}}$$

$$d\,\Bigg(\mathrm{Cosh}\Big[\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}\Big]\,\mathrm{CoshIntegral}\Big[\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}+d\,x\Big]-\mathrm{Sinh}\Big[\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}\Big]\,\mathrm{SinhIntegral}\Big[\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}+d\,x\Big]\Bigg)\Bigg)+$$

$$\Bigg(\mathbb{i}\,\Big(2\,b^{1/3}-3\,(-1)^{1/3}\,b^{1/3}+3\,(-1)^{2/3}\,b^{1/3}\Big)\,\Bigg(\mathrm{CosIntegral}\Big[-\frac{(-1)^{1/6}\,a^{1/3}\,d}{b^{1/3}}+\mathbb{i}\,d\,x\Big]\,\mathrm{Sin}\Big[\frac{(-1)^{1/6}\,a^{1/3}\,d}{b^{1/3}}\Big]-\mathrm{Cos}\Big[\frac{(-1)^{1/6}\,a^{1/3}\,d}{b^{1/3}}\Big]\,\mathrm{SinIntegral}\Big[$$

$$\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} - \mathrm{i}\, d\, x\Big]\Big)\Big) \Big/ \Big(\Big((-1 + (-1)^{1/3})\, (1 + (-1)^{1/3})^2\, a^2\, b^{1/3}\Big) + \frac{1}{(1 + (-1)^{1/3})^2\, a^2\, b^{1/3}}\, \mathrm{i}\, \Big(3\, b^{1/3} - 2\, (-1)^{1/3}\, b^{1/3} + 3\, (-1)^{2/3}\, b^{1/3}\Big)$$

$$\Big(\mathrm{CosIntegral}\Big[-\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} + \mathrm{i}\, d\, x\Big]\, \mathrm{Sin}\Big[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}}\Big] - \mathrm{Cos}\Big[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}}\Big]\, \mathrm{SinIntegral}\Big[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} - \mathrm{i}\, d\, x\Big]\Big)\Big) -$$

$$\mathrm{Sinh}[c]\, \Bigg(\frac{\mathrm{CoshIntegral}[d\, x]}{a^2} + \Bigg(\Big(22 - 21\, (-1)^{1/3} + 21\, (-1)^{2/3}\Big)\, \Bigg(-\frac{b^{1/3}\, \mathrm{Cosh}[d\, x]}{a^{1/3} + b^{1/3}\, x} - d\, \mathrm{CoshIntegral}\Big[d\, \Big(\frac{a^{1/3}}{b^{1/3}} + x\Big)\Big]\, \mathrm{Sinh}\Big[\frac{a^{1/3} d}{b^{1/3}}\Big] +$$

$$d\, \mathrm{Cosh}\Big[\frac{a^{1/3} d}{b^{1/3}}\Big]\, \mathrm{SinhIntegral}\Big[d\, \Big(\frac{a^{1/3}}{b^{1/3}} + x\Big)\Big]\Big]\Big)\Big) \Big/ \Big(3\, (-1 + (-1)^{1/3})\, (1 + (-1)^{1/3})^2\, a^{5/3}\, b^{1/3}\Big) +$$

$$\Bigg(\Big(21 - 22\, (-1)^{1/3} + 21\, (-1)^{2/3}\Big)\, b^{1/3}\, \Bigg(\frac{\mathrm{Cosh}[d\, x]}{b^{1/3}\, \big((-1)^{1/3}\, a^{1/3} - b^{1/3}\, x\big)} + \frac{1}{b^{2/3}} d\, \Bigg(\mathrm{CoshIntegral}\Big[d\, \Big(-\frac{(-1)^{1/3}\, a^{1/3}}{b^{1/3}} + x\Big)\Big]$$

$$\mathrm{Sinh}\Big[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\Big] - \mathrm{Cosh}\Big[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\Big]\, \mathrm{SinhIntegral}\Big[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d\, x\Big]\Bigg)\Bigg)\Bigg) \Big/ \Big(3\, (-1 + (-1)^{1/3})\, (1 + (-1)^{1/3})^2\, a^{5/3}\Big) -$$

$$\Bigg(\Big(2\, b^{1/3} - 3\, (-1)^{1/3}\, b^{1/3} + 3\, (-1)^{2/3}\, b^{1/3}\Big)\, \Big(\mathrm{Cosh}\Big[\frac{a^{1/3} d}{b^{1/3}}\Big]\, \mathrm{CoshIntegral}\Big[\frac{a^{1/3} d}{b^{1/3}} + d\, x\Big] - \mathrm{Sinh}\Big[\frac{a^{1/3} d}{b^{1/3}}\Big]\, \mathrm{SinhIntegral}\Big[\frac{a^{1/3} d}{b^{1/3}} + d\, x\Big]\Big)\Bigg) \Big/$$

$$\Bigg(\Big((-1 + (-1)^{1/3})\, (1 + (-1)^{1/3})^2\, a^2\, b^{1/3}\Big) + \frac{1}{3\, (1 + (-1)^{1/3})^2\, a^{5/3}} \Big(22\, b^{1/3} - 21\, (-1)^{1/3}\, b^{1/3} + 21\, (-1)^{2/3}\, b^{1/3}\Big)\, \Bigg(-\frac{\mathrm{Cosh}[d\, x]}{b^{1/3}\, \big((-1)^{2/3}\, a^{1/3} + b^{1/3}\, x\big)} -$$

$$\frac{1}{b^{2/3}} d\, \Big(\mathrm{CoshIntegral}\Big[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d\, x\Big]\, \mathrm{Sinh}\Big[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\Big] - \mathrm{Cosh}\Big[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\Big]\, \mathrm{SinhIntegral}\Big[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d\, x\Big]\Big)\Bigg) -$$

$$\Bigg(\Big(2\, b^{1/3} - 3\, (-1)^{1/3}\, b^{1/3} + 3\, (-1)^{2/3}\, b^{1/3}\Big)\, \Bigg(\mathrm{Cos}\Big[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}}\Big]\, \mathrm{CosIntegral}\Big[-\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} + \mathrm{i}\, d\, x\Big] + \mathrm{Sin}\Big[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}}\Big]\, \mathrm{SinIntegral}\Big[$$

$$\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} - \mathrm{i}\, d\, x\Big]\Big)\Big) \Big/ \Big(\Big((-1 + (-1)^{1/3})\, (1 + (-1)^{1/3})^2\, a^2\, b^{1/3}\Big) - \frac{1}{(1 + (-1)^{1/3})^2\, a^2\, b^{1/3}} \Big(3\, b^{1/3} - 2\, (-1)^{1/3}\, b^{1/3} + 3\, (-1)^{2/3}\, b^{1/3}\Big)$$

$$\Big(\mathrm{Cos}\Big[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}}\Big]\, \mathrm{CosIntegral}\Big[-\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} + \mathrm{i}\, d\, x\Big] + \mathrm{Sin}\Big[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}}\Big]\, \mathrm{SinIntegral}\Big[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} - \mathrm{i}\, d\, x\Big]\Big)\Big)\Bigg) +$$

$$\frac{1}{2} \Bigg(\mathrm{Cosh}[c]\, \Bigg(\frac{\mathrm{SinhIntegral}[d\, x]}{a^2} - \Bigg(\Big(2\, b^{1/3} - 3\, (-1)^{1/3}\, b^{1/3} + 3\, (-1)^{2/3}\, b^{1/3}\Big)$$

$$\left(-\text{CoshIntegral}\left[d\left(\frac{a^{1/3}}{b^{1/3}}+x\right)\right]\text{Sinh}\left[\frac{a^{1/3}d}{b^{1/3}}\right]+\text{Cosh}\left[\frac{a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[d\left(\frac{a^{1/3}}{b^{1/3}}+x\right)\right]\right)\Big)\Big/$$

$$\left(\left(-1+(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^2b^{1/3}\right)+\left(\left(21-22(-1)^{1/3}+21(-1)^{2/3}\right)b^{1/3}\left(-\frac{\text{Sinh}[d\,x]}{b^{1/3}\left(-(-1)^{1/3}a^{1/3}+b^{1/3}x\right)}+\frac{1}{b^{2/3}}\right.\right.$$

$$\left.\left.d\left(\text{Cosh}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\text{CoshIntegral}\left[-\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}+d\,x\right]-\text{Sinh}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]\right)\right)\right)\Big/$$

$$\left(3\left(-1+(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^{5/3}\right)+\left(\left(22-21(-1)^{1/3}+21(-1)^{2/3}\right)b^{1/3}\left(-\frac{\text{Sinh}[d\,x]}{b^{1/3}\left(a^{1/3}+b^{1/3}x\right)}+\right.\right.$$

$$\left.\left.\frac{d\left(\text{Cosh}\left[\frac{a^{1/3}d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]-\text{Sinh}\left[\frac{a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]\right)}{b^{2/3}}\right)\right)\Big/\left(3\left(-1+(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^{5/3}\right)+$$

$$\frac{1}{3\left(1+(-1)^{1/3}\right)^2a^{5/3}}\left(22\,b^{1/3}-21(-1)^{1/3}b^{1/3}+21(-1)^{2/3}b^{1/3}\right)\left(-\frac{\text{Sinh}[d\,x]}{b^{1/3}\left((-1)^{2/3}a^{1/3}+b^{1/3}x\right)}+\frac{1}{b^{2/3}}\right.$$

$$\left.d\left(\text{Cosh}\left[\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}+d\,x\right]-\text{Sinh}\left[\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}+d\,x\right]\right)\right)+$$

$$\left(i\left(2\,b^{1/3}-3(-1)^{1/3}b^{1/3}+3(-1)^{2/3}b^{1/3}\right)\left(\text{CosIntegral}\left[-\frac{(-1)^{1/6}a^{1/3}d}{b^{1/3}}+i\,d\,x\right]\text{Sin}\left[\frac{(-1)^{1/6}a^{1/3}d}{b^{1/3}}\right]-\right.\right.$$

$$\left.\left.\text{Cos}\left[\frac{(-1)^{1/6}a^{1/3}d}{b^{1/3}}\right]\text{SinIntegral}\left[\frac{(-1)^{1/6}a^{1/3}d}{b^{1/3}}-i\,d\,x\right]\right)\right)\Big/$$

$$\left(\left(-1+(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^2b^{1/3}\right)+\frac{1}{\left(1+(-1)^{1/3}\right)^2a^2b^{1/3}}i\left(3\,b^{1/3}-2(-1)^{1/3}b^{1/3}+3(-1)^{2/3}b^{1/3}\right)$$

$$\left(\text{CosIntegral}\left[-\frac{(-1)^{5/6}a^{1/3}d}{b^{1/3}}+i\,d\,x\right]\text{Sin}\left[\frac{(-1)^{5/6}a^{1/3}d}{b^{1/3}}\right]-\text{Cos}\left[\frac{(-1)^{5/6}a^{1/3}d}{b^{1/3}}\right]\text{SinIntegral}\left[\frac{(-1)^{5/6}a^{1/3}d}{b^{1/3}}-i\,d\,x\right]\right)\Bigg)+$$

$$\text{Sinh}[c]\left(\frac{\text{CoshIntegral}[d\,x]}{a^2}+\left(\left(22-21(-1)^{1/3}+21(-1)^{2/3}\right)\left(-\frac{b^{1/3}\text{Cosh}[d\,x]}{a^{1/3}+b^{1/3}x}-d\,\text{CoshIntegral}\left[d\left(\frac{a^{1/3}}{b^{1/3}}+x\right)\right]\text{Sinh}\left[\frac{a^{1/3}d}{b^{1/3}}\right]+\right.\right.\right.$$

$$\left.\left.d\,\text{Cosh}\left[\frac{a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[d\left(\frac{a^{1/3}}{b^{1/3}}+x\right)\right]\right)\right)\Big/\left(3\left(-1+(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^{5/3}b^{1/3}\right)+$$

$$\left(\left(21-22(-1)^{1/3}+21(-1)^{2/3}\right)b^{1/3}\left(\frac{\text{Cosh}[d\,x]}{b^{1/3}\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)}+\frac{1}{b^{2/3}}d\left(\text{CoshIntegral}\left[d\left(-\frac{(-1)^{1/3}a^{1/3}}{b^{1/3}}+x\right)\right]\right.\right.\right.$$

$$\left.\left.\left.\text{Sinh}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]-\text{Cosh}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{1/3}a^{1/3}d}{b^{1/3}}-d\,x\right]\right)\right)\right)\Big/\left(3\left(-1+(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^{5/3}\right)-$$

$$\left(\left(2b^{1/3}-3(-1)^{1/3}b^{1/3}+3(-1)^{2/3}b^{1/3}\right)\left(\text{Cosh}\left[\frac{a^{1/3}d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]-\text{Sinh}\left[\frac{a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{a^{1/3}d}{b^{1/3}}+d\,x\right]\right)\right)\Big/$$

$$\left(\left(-1+(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^2b^{1/3}\right)+\frac{1}{3\left(1+(-1)^{1/3}\right)^2a^{5/3}}\left(22b^{1/3}-21(-1)^{1/3}b^{1/3}+21(-1)^{2/3}b^{1/3}\right)\left(-\frac{\text{Cosh}[d\,x]}{b^{1/3}\left((-1)^{2/3}a^{1/3}+b^{1/3}x\right)}-\right.$$

$$\left.\frac{1}{b^{2/3}}d\left(\text{CoshIntegral}\left[\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}+d\,x\right]\text{Sinh}\left[\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]-\text{Cosh}\left[\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{2/3}a^{1/3}d}{b^{1/3}}+d\,x\right]\right)\right)-$$

$$\left(\left(2b^{1/3}-3(-1)^{1/3}b^{1/3}+3(-1)^{2/3}b^{1/3}\right)\left(\text{Cos}\left[\frac{(-1)^{1/6}a^{1/3}d}{b^{1/3}}\right]\text{CosIntegral}\left[-\frac{(-1)^{1/6}a^{1/3}d}{b^{1/3}}+i\,d\,x\right]+\right.\right.$$

$$\left.\left.\text{Sin}\left[\frac{(-1)^{1/6}a^{1/3}d}{b^{1/3}}\right]\text{SinIntegral}\left[\frac{(-1)^{1/6}a^{1/3}d}{b^{1/3}}-i\,d\,x\right]\right)\right)\Big/$$

$$\left(\left(-1+(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^2b^{1/3}\right)-\frac{1}{\left(1+(-1)^{1/3}\right)^2a^2b^{1/3}}\left(3b^{1/3}-2(-1)^{1/3}b^{1/3}+3(-1)^{2/3}b^{1/3}\right)$$

$$\left.\left.\left(\text{Cos}\left[\frac{(-1)^{5/6}a^{1/3}d}{b^{1/3}}\right]\text{CosIntegral}\left[-\frac{(-1)^{5/6}a^{1/3}d}{b^{1/3}}+i\,d\,x\right]+\text{Sin}\left[\frac{(-1)^{5/6}a^{1/3}d}{b^{1/3}}\right]\text{SinIntegral}\left[\frac{(-1)^{5/6}a^{1/3}d}{b^{1/3}}-i\,d\,x\right]\right)\right)\right)$$

## Problem 107: Result is not expressed in closed-form.

$$\int\frac{x^5\,\text{Cosh}[c+d\,x]}{\left(a+b\,x^3\right)^3}\,dx$$

Optimal (type 4, 784 leaves, 36 steps) :

$$-\frac{x^3 \text{Cosh}[c+d x]}{6 b \left(a+b x^3\right)^2}-\frac{\text{Cosh}[c+d x]}{6 b^2 \left(a+b x^3\right)}-\frac{(-1)^{2/3} d^2 \text{Cosh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{54 a^{1/3} b^{8/3}}+$$
$$\frac{(-1)^{1/3} d^2 \text{Cosh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}-d x\right]}{54 a^{1/3} b^{8/3}}-\frac{d^2 \text{Cosh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \text{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{54 a^{1/3} b^{8/3}}+$$
$$\frac{2 d \text{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right] \text{Sinh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right]}{27 a^{2/3} b^{7/3}}-\frac{2 (-1)^{1/3} d \text{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right] \text{Sinh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{27 a^{2/3} b^{7/3}}+$$
$$\frac{2 (-1)^{2/3} d \text{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}-d x\right] \text{Sinh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{27 a^{2/3} b^{7/3}}-\frac{d x \text{Sinh}[c+d x]}{18 b^2 \left(a+b x^3\right)}+$$
$$\frac{2 (-1)^{1/3} d \text{Cosh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{27 a^{2/3} b^{7/3}}+\frac{(-1)^{2/3} d^2 \text{Sinh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{54 a^{1/3} b^{8/3}}+$$
$$\frac{2 d \text{Cosh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{27 a^{2/3} b^{7/3}}-\frac{d^2 \text{Sinh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{54 a^{1/3} b^{8/3}}+$$
$$\frac{2 (-1)^{2/3} d \text{Cosh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}+d x\right]}{27 a^{2/3} b^{7/3}}+\frac{(-1)^{1/3} d^2 \text{Sinh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}+d x\right]}{54 a^{1/3} b^{8/3}}$$

Result (type 7, 397 leaves):

$$\frac{1}{108 b^3}\Bigg(d \text{RootSum}\Big[a+b \#1^3 \&,$$
$$\frac{1}{\#1^2}\big(-4 \text{Cosh}[c+d \#1] \text{CoshIntegral}[d (x-\#1)]+4 \text{CoshIntegral}[d (x-\#1)] \text{Sinh}[c+d \#1]+4 \text{Cosh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)]-$$
$$4 \text{Sinh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)]+d \text{Cosh}[c+d \#1] \text{CoshIntegral}[d (x-\#1)] \#1-d \text{CoshIntegral}[d (x-\#1)] \text{Sinh}[c+d \#1] \#1-$$
$$d \text{Cosh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)] \#1+d \text{Sinh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)] \#1\big) \&\Big]+$$
$$d \text{RootSum}\Big[a+b \#1^3 \&, \frac{1}{\#1^2}\big(4 \text{Cosh}[c+d \#1] \text{CoshIntegral}[d (x-\#1)]+4 \text{CoshIntegral}[d (x-\#1)] \text{Sinh}[c+d \#1]+$$
$$4 \text{Cosh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)]+4 \text{Sinh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)]+d \text{Cosh}[c+d \#1] \text{CoshIntegral}[d (x-\#1)] \#1+$$
$$d \text{CoshIntegral}[d (x-\#1)] \text{Sinh}[c+d \#1] \#1+d \text{Cosh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)] \#1+$$
$$d \text{Sinh}[c+d \#1] \text{SinhIntegral}[d (x-\#1)] \#1\big) \&\Big]-\frac{6 b \left(3 \left(a+2 b x^3\right) \text{Cosh}[c+d x]+d x \left(a+b x^3\right) \text{Sinh}[c+d x]\right)}{\left(a+b x^3\right)^2}\Bigg)$$

## Problem 108: Result is not expressed in closed-form.

$$\int \frac{x^4 \operatorname{Cosh}[c+d x]}{\left(a+b x^3\right)^3} \, dx$$

Optimal (type 4, 1105 leaves, 47 steps):

$$\frac{\operatorname{Cosh}[c+d x]}{9 a b^2 x}-\frac{x^2 \operatorname{Cosh}[c+d x]}{6 b \left(a+b x^3\right)^2}-\frac{\operatorname{Cosh}[c+d x]}{9 b^2 x \left(a+b x^3\right)}-\frac{(-1)^{2/3} \operatorname{Cosh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{27 a^{4/3} b^{5/3}}-$$
$$\frac{(-1)^{1/3} d^2 \operatorname{Cosh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{54 a^{2/3} b^{7/3}}+\frac{(-1)^{1/3} \operatorname{Cosh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}-d x\right]}{27 a^{4/3} b^{5/3}}+$$
$$\frac{(-1)^{2/3} d^2 \operatorname{Cosh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}-d x\right]}{54 a^{2/3} b^{7/3}}-\frac{\operatorname{Cosh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{27 a^{4/3} b^{5/3}}+$$
$$\frac{d^2 \operatorname{Cosh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{54 a^{2/3} b^{7/3}}-\frac{d \operatorname{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right] \operatorname{Sinh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right]}{27 a b^2}-$$
$$\frac{d \operatorname{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right] \operatorname{Sinh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{27 a b^2}-\frac{d \operatorname{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}-d x\right] \operatorname{Sinh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{27 a b^2}-\frac{d \operatorname{Sinh}[c+d x]}{18 b^2 \left(a+b x^3\right)}+$$
$$\frac{d \operatorname{Cosh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{27 a b^2}+\frac{(-1)^{2/3} \operatorname{Sinh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{27 a^{4/3} b^{5/3}}+$$
$$\frac{(-1)^{1/3} d^2 \operatorname{Sinh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{54 a^{2/3} b^{7/3}}-\frac{d \operatorname{Cosh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{27 a b^2}-$$
$$\frac{\operatorname{Sinh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{27 a^{4/3} b^{5/3}}+\frac{d^2 \operatorname{Sinh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{54 a^{2/3} b^{7/3}}-\frac{d \operatorname{Cosh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}+d x\right]}{27 a b^2}+$$
$$\frac{(-1)^{1/3} \operatorname{Sinh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}+d x\right]}{27 a^{4/3} b^{5/3}}+\frac{(-1)^{2/3} d^2 \operatorname{Sinh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}+d x\right]}{54 a^{2/3} b^{7/3}}$$

Result (type 7, 675 leaves):

$$\frac{1}{108\,a\,b^3}\left(\text{RootSum}\left[a+b\,\#1^3\,\&,\ \frac{1}{\#1^2}\left(a\,d^2\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]-a\,d^2\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]-a\,d^2\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]+a\,d^2\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]+2\,b\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]\,\#1-2\,b\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]\,\#1-2\,b\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1+2\,b\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1+2\,b\,d\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]\,\#1^2-2\,b\,d\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]\,\#1^2-2\,b\,d\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1^2+2\,b\,d\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1^2\right)\&\right]-\text{RootSum}\left[a+b\,\#1^3\,\&,\ \frac{1}{\#1^2}\left(-a\,d^2\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]-a\,d^2\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]-a\,d^2\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]-a\,d^2\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]-2\,b\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]\,\#1-2\,b\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]\,\#1-2\,b\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1-2\,b\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1+2\,b\,d\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]\,\#1^2+2\,b\,d\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]\,\#1^2+2\,b\,d\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1^2+2\,b\,d\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1^2\right)\&\right]+\frac{6\,b\,\text{Cosh}[d\,x]\,\left(b\,x^2\,\left(-a+2\,b\,x^3\right)\,\text{Cosh}[c]-a\,d\,\left(a+b\,x^3\right)\,\text{Sinh}[c]\right)}{\left(a+b\,x^3\right)^2}+\frac{6\,b\,\left(-a\,d\,\left(a+b\,x^3\right)\,\text{Cosh}[c]+b\,x^2\,\left(-a+2\,b\,x^3\right)\,\text{Sinh}[c]\right)\,\text{Sinh}[d\,x]}{\left(a+b\,x^3\right)^2}\right)$$

## Problem 109: Result is not expressed in closed-form.

$$\int\frac{x^3\,\text{Cosh}[c+d\,x]}{\left(a+b\,x^3\right)^3}\,dx$$

Optimal (type 4, 776 leaves, 71 steps):

$$\frac{\text{Cosh}[c+d x]}{18 a b^2 x^2}-\frac{x\,\text{Cosh}[c+d x]}{6 b\left(a+b x^3\right)^2}-\frac{\text{Cosh}[c+d x]}{18 b^2 x^2\left(a+b x^3\right)}-\frac{(-1)^{1/3}\text{Cosh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{27 a^{5/3} b^{4/3}}-$$
$$\frac{d^2\,\text{Cosh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{54 a b^2}+\frac{(-1)^{2/3}\text{Cosh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]\text{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}-d x\right]}{27 a^{5/3} b^{4/3}}-$$
$$\frac{d^2\,\text{Cosh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]\text{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}-d x\right]}{54 a b^2}+\frac{\text{Cosh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{27 a^{5/3} b^{4/3}}-$$
$$\frac{d^2\,\text{Cosh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right]\text{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{54 a b^2}+\frac{d\,\text{Sinh}[c+d x]}{18 a b^2 x}-\frac{d\,\text{Sinh}[c+d x]}{18 b^2 x\left(a+b x^3\right)}+\frac{(-1)^{1/3}\text{Sinh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{27 a^{5/3} b^{4/3}}+$$
$$\frac{d^2\,\text{Sinh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{54 a b^2}+\frac{\text{Sinh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{27 a^{5/3} b^{4/3}}-\frac{d^2\,\text{Sinh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{54 a b^2}+$$
$$\frac{(-1)^{2/3}\text{Sinh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}+d x\right]}{27 a^{5/3} b^{4/3}}-\frac{d^2\,\text{Sinh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]\text{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}+d x\right]}{54 a b^2}$$

Result (type 7, 429 leaves):

$$-\frac{1}{108 a b^2}\Bigg(\text{RootSum}\Big[a+b\,\#1^3\,\&,$$
$$\frac{1}{\#1^2}\big(-2\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]+2\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]+2\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]-$$
$$2\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]+d^2\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]\,\#1^2-d^2\,\text{CoshIntegral}[d\,(x-\#1)]$$
$$\text{Sinh}[c+d\,\#1]\,\#1^2-d^2\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1^2+d^2\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1^2\big)\,\&\Big]+$$
$$\text{RootSum}\Big[a+b\,\#1^3\,\&,\ \frac{1}{\#1^2}\big(-2\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]-2\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]-$$
$$2\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]-2\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]+d^2\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]\,\#1^2+$$
$$d^2\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]\,\#1^2+d^2\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1^2+$$
$$d^2\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1^2\big)\,\&\Big]-\frac{6 b x\left(\left(-2 a+b x^3\right)\text{Cosh}[c+d x]+d x\left(a+b x^3\right)\text{Sinh}[c+d x]\right)}{\left(a+b x^3\right)^2}\Bigg)$$

## Problem 110: Result is not expressed in closed-form.

$$\int\frac{x^2\,\text{Cosh}[c+d x]}{\left(a+b x^3\right)^3}\,dx$$

Optimal (type 4, 781 leaves, 37 steps):

$$-\frac{\text{Cosh}\left[c+d\,x\right]}{6\,b\,\left(a+b\,x^3\right)^2}+\frac{(-1)^{2/3}\,d^2\,\text{Cosh}\left[c+\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}\right]\,\text{CoshIntegral}\left[\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}-d\,x\right]}{54\,a^{4/3}\,b^{5/3}}-$$

$$\frac{(-1)^{1/3}\,d^2\,\text{Cosh}\left[c-\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}\right]\,\text{CoshIntegral}\left[-\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}-d\,x\right]}{54\,a^{4/3}\,b^{5/3}}+\frac{d^2\,\text{Cosh}\left[c-\frac{a^{1/3}\,d}{b^{1/3}}\right]\,\text{CoshIntegral}\left[\frac{a^{1/3}\,d}{b^{1/3}}+d\,x\right]}{54\,a^{4/3}\,b^{5/3}}+$$

$$\frac{d\,\text{CoshIntegral}\left[\frac{a^{1/3}\,d}{b^{1/3}}+d\,x\right]\,\text{Sinh}\left[c-\frac{a^{1/3}\,d}{b^{1/3}}\right]}{27\,a^{5/3}\,b^{4/3}}-\frac{(-1)^{1/3}\,d\,\text{CoshIntegral}\left[\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}-d\,x\right]\,\text{Sinh}\left[c+\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}\right]}{27\,a^{5/3}\,b^{4/3}}+$$

$$\frac{(-1)^{2/3}\,d\,\text{CoshIntegral}\left[-\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}-d\,x\right]\,\text{Sinh}\left[c-\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}\right]}{27\,a^{5/3}\,b^{4/3}}+\frac{d\,\text{Sinh}\left[c+d\,x\right]}{18\,a\,b^2\,x^2}-\frac{d\,\text{Sinh}\left[c+d\,x\right]}{18\,b^2\,x^2\,\left(a+b\,x^3\right)}+$$

$$\frac{(-1)^{1/3}\,d\,\text{Cosh}\left[c+\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}\right]\,\text{SinhIntegral}\left[\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}-d\,x\right]}{27\,a^{5/3}\,b^{4/3}}-\frac{(-1)^{2/3}\,d^2\,\text{Sinh}\left[c+\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}\right]\,\text{SinhIntegral}\left[\frac{(-1)^{1/3}\,a^{1/3}\,d}{b^{1/3}}-d\,x\right]}{54\,a^{4/3}\,b^{5/3}}+$$

$$\frac{d\,\text{Cosh}\left[c-\frac{a^{1/3}\,d}{b^{1/3}}\right]\,\text{SinhIntegral}\left[\frac{a^{1/3}\,d}{b^{1/3}}+d\,x\right]}{27\,a^{5/3}\,b^{4/3}}+\frac{d^2\,\text{Sinh}\left[c-\frac{a^{1/3}\,d}{b^{1/3}}\right]\,\text{SinhIntegral}\left[\frac{a^{1/3}\,d}{b^{1/3}}+d\,x\right]}{54\,a^{4/3}\,b^{5/3}}+$$

$$\frac{(-1)^{2/3}\,d\,\text{Cosh}\left[c-\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}\right]\,\text{SinhIntegral}\left[\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}+d\,x\right]}{27\,a^{5/3}\,b^{4/3}}-\frac{(-1)^{1/3}\,d^2\,\text{Sinh}\left[c-\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}\right]\,\text{SinhIntegral}\left[\frac{(-1)^{2/3}\,a^{1/3}\,d}{b^{1/3}}+d\,x\right]}{54\,a^{4/3}\,b^{5/3}}$$

Result (type 7, 423 leaves) :

$$-\frac{1}{108\,a\,b^2}\Bigg(d\,\text{RootSum}\Big[a+b\,\#1^3\,\&,$$

$$\frac{1}{\#1^2}\big(2\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]-2\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]-2\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]+$$

$$2\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]+d\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]\,\#1-d\,\text{CoshIntegral}[d\,(x-\#1)]$$

$$\text{Sinh}[c+d\,\#1]\,\#1-d\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1+d\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1\big)\,\&\Big]+$$

$$d\,\text{RootSum}\Big[a+b\,\#1^3\,\&,\,\frac{1}{\#1^2}\big(-2\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]-2\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]-$$

$$2\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]-2\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]+$$

$$d\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]\,\#1+d\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]\,\#1+$$

$$d\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1+d\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1\big)\,\&\Big]-$$

$$\frac{6\,b\,\text{Cosh}[d\,x]\,\left(-3\,a\,\text{Cosh}[c]+d\,x\,\left(a+b\,x^3\right)\,\text{Sinh}[c]\right)}{\left(a+b\,x^3\right)^2}-\frac{6\,b\,\left(d\,x\,\left(a+b\,x^3\right)\,\text{Cosh}[c]-3\,a\,\text{Sinh}[c]\right)\,\text{Sinh}[d\,x]}{\left(a+b\,x^3\right)^2}\Bigg)$$

## Problem 111: Result is not expressed in closed-form.

$$\int \frac{x \operatorname{Cosh}[c+d x]}{\left(a+b x^3\right)^3} \, dx$$

Optimal (type 4, 1147 leaves, 89 steps):

$$-\frac{\operatorname{Cosh}[c+d x]}{18 a b^2 x^4}+\frac{2 \operatorname{Cosh}[c+d x]}{9 a^2 b x}-\frac{\operatorname{Cosh}[c+d x]}{6 b x \left(a+b x^3\right)^2}+\frac{\operatorname{Cosh}[c+d x]}{18 b^2 x^4 \left(a+b x^3\right)}-\frac{2 (-1)^{2/3} \operatorname{Cosh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{27 a^{7/3} b^{2/3}}+$$

$$\frac{(-1)^{1/3} d^2 \operatorname{Cosh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{54 a^{5/3} b^{4/3}}+\frac{2 (-1)^{1/3} \operatorname{Cosh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}-d x\right]}{27 a^{7/3} b^{2/3}}-$$

$$\frac{(-1)^{2/3} d^2 \operatorname{Cosh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}-d x\right]}{54 a^{5/3} b^{4/3}}-\frac{2 \operatorname{Cosh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{27 a^{7/3} b^{2/3}}-$$

$$\frac{d^2 \operatorname{Cosh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{54 a^{5/3} b^{4/3}}-\frac{2 d \operatorname{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right] \operatorname{Sinh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right]}{27 a^2 b}-$$

$$\frac{2 d \operatorname{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right] \operatorname{Sinh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{27 a^2 b}-\frac{2 d \operatorname{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}-d x\right] \operatorname{Sinh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{27 a^2 b}+$$

$$\frac{d \operatorname{Sinh}[c+d x]}{18 a b^2 x^3}-\frac{d \operatorname{Sinh}[c+d x]}{18 b^2 x^3 \left(a+b x^3\right)}+\frac{2 d \operatorname{Cosh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{27 a^2 b}+$$

$$\frac{2 (-1)^{2/3} \operatorname{Sinh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{27 a^{7/3} b^{2/3}}-\frac{(-1)^{1/3} d^2 \operatorname{Sinh}\left[c+\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}-d x\right]}{54 a^{5/3} b^{4/3}}-$$

$$\frac{2 d \operatorname{Cosh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{27 a^2 b}-\frac{2 \operatorname{Sinh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{27 a^{7/3} b^{2/3}}-$$

$$\frac{d^2 \operatorname{Sinh}\left[c-\frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}}+d x\right]}{54 a^{5/3} b^{4/3}}-\frac{2 d \operatorname{Cosh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}+d x\right]}{27 a^2 b}+$$

$$\frac{2 (-1)^{1/3} \operatorname{Sinh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}+d x\right]}{27 a^{7/3} b^{2/3}}-\frac{(-1)^{2/3} d^2 \operatorname{Sinh}\left[c-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}+d x\right]}{54 a^{5/3} b^{4/3}}$$

Result (type 7, 669 leaves):

$$\frac{1}{108\,a^2\,b^2}\Bigg(\text{RootSum}\Big[a+b\,\#1^3\,\&,\ \frac{1}{\#1^2}\Big(-a\,d^2\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]+a\,d^2\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]+a\,d^2\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]-a\,d^2\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]+4\,b\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]\,\#1-4\,b\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]\,\#1-4\,b\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1+4\,b\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1+4\,b\,d\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]\,\#1^2-4\,b\,d\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]\,\#1^2-4\,b\,d\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1^2+4\,b\,d\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1^2\Big)\,\&\Big]-\text{RootSum}\Big[a+b\,\#1^3\,\&,\ \frac{1}{\#1^2}\Big(a\,d^2\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]+a\,d^2\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]+a\,d^2\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]+a\,d^2\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]-4\,b\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]\,\#1-4\,b\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]\,\#1-4\,b\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1-4\,b\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1+4\,b\,d\,\text{Cosh}[c+d\,\#1]\,\text{CoshIntegral}[d\,(x-\#1)]\,\#1^2+4\,b\,d\,\text{CoshIntegral}[d\,(x-\#1)]\,\text{Sinh}[c+d\,\#1]\,\#1^2+4\,b\,d\,\text{Cosh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1^2+4\,b\,d\,\text{Sinh}[c+d\,\#1]\,\text{SinhIntegral}[d\,(x-\#1)]\,\#1^2\Big)\,\&\Big]+\frac{6\,b\,\text{Cosh}[d\,x]\,\left(b\,x^2\,(7\,a+4\,b\,x^3)\,\text{Cosh}[c]+a\,d\,(a+b\,x^3)\,\text{Sinh}[c]\right)}{(a+b\,x^3)^2}+\frac{6\,b\,\left(a\,d\,(a+b\,x^3)\,\text{Cosh}[c]+b\,x^2\,(7\,a+4\,b\,x^3)\,\text{Sinh}[c]\right)\,\text{Sinh}[d\,x]}{(a+b\,x^3)^2}\Bigg)$$

# Test results for the 68 problems in "6.2.3 (e x)^m (a+b cosh(c+d x^n))^p.m"

## Problem 3: Result more than twice size of optimal antiderivative.

$$\int x\,\text{Cosh}[a+b\,x^2]\,dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{\text{Sinh}[a+b\,x^2]}{2\,b}$$

Result (type 3, 31 leaves):

$$\frac{\text{Cosh}[b\,x^2]\,\text{Sinh}[a]}{2\,b}+\frac{\text{Cosh}[a]\,\text{Sinh}[b\,x^2]}{2\,b}$$

## Problem 67: Result is not expressed in closed-form.

$$\int \frac{\text{Cosh}\left[a+b\,(c+d\,x)^{1/3}\right]}{x}\,dx$$

Optimal (type 4, 232 leaves, 13 steps):

$$\mathrm{Cosh}\left[a+b\,c^{1/3}\right]\mathrm{CoshIntegral}\left[b\left(c^{1/3}-(c+d\,x)^{1/3}\right)\right]+\mathrm{Cosh}\left[a+(-1)^{2/3}b\,c^{1/3}\right]\mathrm{CoshIntegral}\left[-b\left((-1)^{2/3}c^{1/3}-(c+d\,x)^{1/3}\right)\right]+\mathrm{Cosh}\left[a-(-1)^{1/3}b\,c^{1/3}\right]\mathrm{CoshIntegral}\left[b\left((-1)^{1/3}c^{1/3}+(c+d\,x)^{1/3}\right)\right]-\mathrm{Sinh}\left[a+b\,c^{1/3}\right]\mathrm{SinhIntegral}\left[b\left(c^{1/3}-(c+d\,x)^{1/3}\right)\right]-\mathrm{Sinh}\left[a+(-1)^{2/3}b\,c^{1/3}\right]\mathrm{SinhIntegral}\left[b\left((-1)^{2/3}c^{1/3}-(c+d\,x)^{1/3}\right)\right]+\mathrm{Sinh}\left[a-(-1)^{1/3}b\,c^{1/3}\right]\mathrm{SinhIntegral}\left[b\left((-1)^{1/3}c^{1/3}+(c+d\,x)^{1/3}\right)\right]$$

Result (type 7, 231 leaves):

$$\frac{1}{2}\left(\mathrm{RootSum}\left[c-\#1^3\,\&,\ \mathrm{Cosh}[a+b\,\#1]\,\mathrm{CoshIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]-\mathrm{CoshIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]\mathrm{Sinh}[a+b\,\#1]-\mathrm{Cosh}[a+b\,\#1]\,\mathrm{SinhIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]+\mathrm{Sinh}[a+b\,\#1]\,\mathrm{SinhIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]\,\&\right]+\mathrm{RootSum}\left[c-\#1^3\,\&,\ \mathrm{Cosh}[a+b\,\#1]\,\mathrm{CoshIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]+\mathrm{CoshIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]\mathrm{Sinh}[a+b\,\#1]+\mathrm{Cosh}[a+b\,\#1]\,\mathrm{SinhIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]+\mathrm{Sinh}[a+b\,\#1]\,\mathrm{SinhIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]\,\&\right]\right)$$

## Problem 68: Result is not expressed in closed-form.

$$\int\frac{\mathrm{Cosh}\left[a+b\,(c+d\,x)^{1/3}\right]}{x^2}\,dx$$

Optimal (type 4, 329 leaves, 14 steps):

$$-\frac{\mathrm{Cosh}\left[a+b\,(c+d\,x)^{1/3}\right]}{x}+\frac{b\,d\,\mathrm{CoshIntegral}\left[b\left(c^{1/3}-(c+d\,x)^{1/3}\right)\right]\mathrm{Sinh}\left[a+b\,c^{1/3}\right]}{3\,c^{2/3}}-\frac{(-1)^{1/3}b\,d\,\mathrm{CoshIntegral}\left[b\left((-1)^{1/3}c^{1/3}+(c+d\,x)^{1/3}\right)\right]\mathrm{Sinh}\left[a-(-1)^{1/3}b\,c^{1/3}\right]}{3\,c^{2/3}}+\frac{(-1)^{2/3}b\,d\,\mathrm{CoshIntegral}\left[-b\left((-1)^{2/3}c^{1/3}-(c+d\,x)^{1/3}\right)\right]\mathrm{Sinh}\left[a+(-1)^{2/3}b\,c^{1/3}\right]}{3\,c^{2/3}}-\frac{b\,d\,\mathrm{Cosh}\left[a+b\,c^{1/3}\right]\mathrm{SinhIntegral}\left[b\left(c^{1/3}-(c+d\,x)^{1/3}\right)\right]}{3\,c^{2/3}}-\frac{(-1)^{2/3}b\,d\,\mathrm{Cosh}\left[a+(-1)^{2/3}b\,c^{1/3}\right]\mathrm{SinhIntegral}\left[b\left((-1)^{2/3}c^{1/3}-(c+d\,x)^{1/3}\right)\right]}{3\,c^{2/3}}-\frac{(-1)^{1/3}b\,d\,\mathrm{Cosh}\left[a-(-1)^{1/3}b\,c^{1/3}\right]\mathrm{SinhIntegral}\left[b\left((-1)^{1/3}c^{1/3}+(c+d\,x)^{1/3}\right)\right]}{3\,c^{2/3}}$$

Result (type 7, 211 leaves):

$$\frac{1}{6x}\Bigg(b\,d\,x\,\text{RootSum}\Big[c-\#1^3\,\&,\ \frac{e^{a+b\,\#1}\,\text{ExpIntegralEi}\big[b\,\big((c+d\,x)^{1/3}-\#1\big)\big]}{\#1^2}\,\&\Big]+e^{-a}\Big(-3\,e^{-b\,(c+d\,x)^{1/3}}\Big(1+e^{2\,\left(a+b\,(c+d\,x)^{1/3}\right)}\Big)-$$
$$b\,d\,x\,\text{RootSum}\Big[c-\#1^3\,\&,\ \frac{1}{\#1^2}\Big(\text{Cosh}[b\,\#1]\,\text{CoshIntegral}\big[b\,\big((c+d\,x)^{1/3}-\#1\big)\big]-\text{CoshIntegral}\big[b\,\big((c+d\,x)^{1/3}-\#1\big)\big]\,\text{Sinh}[b\,\#1]-$$
$$\text{Cosh}[b\,\#1]\,\text{SinhIntegral}\big[b\,\big((c+d\,x)^{1/3}-\#1\big)\big]+\text{Sinh}[b\,\#1]\,\text{SinhIntegral}\big[b\,\big((c+d\,x)^{1/3}-\#1\big)\big]\Big)\,\&\Big]\Big)\Bigg)$$

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# Test results for the 33 problems in "6.2.4 (d+e x)^m cosh(a+b x+c x^2)^n.m"

## Problem 19: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{Cosh}\left[a+b\,x+c\,x^2\right]^2}{x}\,dx$$

Optimal (type 9, 32 leaves, 2 steps):

$$\frac{\text{Log}[x]}{2}+\frac{1}{2}\,\text{Unintegrable}\Big[\frac{\text{Cosh}\left[2\,a+2\,b\,x+2\,c\,x^2\right]}{x},\ x\Big]$$

Result (type 1, 1 leaves):

???

## Problem 23: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{Cosh}\left[a+b\,x-c\,x^2\right]^2}{x}\,dx$$

Optimal (type 9, 32 leaves, 2 steps):

$$\frac{\text{Log}[x]}{2}+\frac{1}{2}\,\text{Unintegrable}\Big[\frac{\text{Cosh}\left[2\,a+2\,b\,x-2\,c\,x^2\right]}{x},\ x\Big]$$

Result (type 1, 1 leaves):

???

# Test results for the 336 problems in "6.2.5 Hyperbolic cosine functions.m"

## Problem 1: Result more than twice size of optimal antiderivative.

$$\int \text{Cosh}[a+b\,x]\,dx$$

Optimal (type 3, 10 leaves, 1 step):

$$\frac{\text{Sinh}[a+b\,x]}{b}$$

Result (type 3, 21 leaves):

$$\frac{\text{Cosh}[b\,x]\,\text{Sinh}[a]}{b}+\frac{\text{Cosh}[a]\,\text{Sinh}[b\,x]}{b}$$

## Problem 75: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{5+3\,\text{Cosh}[c+d\,x]}\,dx$$

Optimal (type 3, 31 leaves, 1 step):

$$\frac{x}{4}-\frac{\text{ArcTanh}\left[\frac{\text{Sinh}[c+d\,x]}{3+\text{Cosh}[c+d\,x]}\right]}{2\,d}$$

Result (type 3, 65 leaves):

$$-\frac{\text{Log}\left[2\,\text{Cosh}\left[\frac{1}{2}\,(c+d\,x)\right]-\text{Sinh}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{4\,d}+\frac{\text{Log}\left[2\,\text{Cosh}\left[\frac{1}{2}\,(c+d\,x)\right]+\text{Sinh}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{4\,d}$$

## Problem 76: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(5+3\,\text{Cosh}[c+d\,x])^2}\,dx$$

Optimal (type 3, 56 leaves, 3 steps):

$$\frac{5\,x}{64}-\frac{5\,\text{ArcTanh}\left[\frac{\text{Sinh}[c+d\,x]}{3+\text{Cosh}[c+d\,x]}\right]}{32\,d}-\frac{3\,\text{Sinh}[c+d\,x]}{16\,d\,(5+3\,\text{Cosh}[c+d\,x])}$$

Result (type 3, 144 leaves):

$$\frac{1}{64\,d\,(5+3\,\mathrm{Cosh}[c+d\,x])}\left(-15\,\mathrm{Cosh}[c+d\,x]\left(\mathrm{Log}\left[2\,\mathrm{Cosh}\left[\frac{1}{2}(c+d\,x)\right]-\mathrm{Sinh}\left[\frac{1}{2}(c+d\,x)\right]\right]-\mathrm{Log}\left[2\,\mathrm{Cosh}\left[\frac{1}{2}(c+d\,x)\right]+\mathrm{Sinh}\left[\frac{1}{2}(c+d\,x)\right]\right]\right)+25\left(-\mathrm{Log}\left[2\,\mathrm{Cosh}\left[\frac{1}{2}(c+d\,x)\right]-\mathrm{Sinh}\left[\frac{1}{2}(c+d\,x)\right]\right]+\mathrm{Log}\left[2\,\mathrm{Cosh}\left[\frac{1}{2}(c+d\,x)\right]+\mathrm{Sinh}\left[\frac{1}{2}(c+d\,x)\right]\right]\right)-12\,\mathrm{Sinh}[c+d\,x]\right)$$

## Problem 77: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{(5+3\,\mathrm{Cosh}[c+d\,x])^3}\,dx$$

Optimal (type 3, 81 leaves, 4 steps):

$$\frac{59\,x}{2048}-\frac{59\,\mathrm{ArcTanh}\left[\frac{\mathrm{Sinh}[c+d\,x]}{3+\mathrm{Cosh}[c+d\,x]}\right]}{1024\,d}-\frac{3\,\mathrm{Sinh}[c+d\,x]}{32\,d\,(5+3\,\mathrm{Cosh}[c+d\,x])^2}-\frac{45\,\mathrm{Sinh}[c+d\,x]}{512\,d\,(5+3\,\mathrm{Cosh}[c+d\,x])}$$

Result (type 3, 217 leaves):

$$-\frac{59\,\mathrm{Log}\left[2\,\mathrm{Cosh}\left[\frac{1}{2}(c+d\,x)\right]-\mathrm{Sinh}\left[\frac{1}{2}(c+d\,x)\right]\right]}{2048\,d}+\frac{59\,\mathrm{Log}\left[2\,\mathrm{Cosh}\left[\frac{1}{2}(c+d\,x)\right]+\mathrm{Sinh}\left[\frac{1}{2}(c+d\,x)\right]\right]}{2048\,d}-\frac{3}{512\,d\left(2\,\mathrm{Cosh}\left[\frac{1}{2}(c+d\,x)\right]-\mathrm{Sinh}\left[\frac{1}{2}(c+d\,x)\right]\right)^2}-\frac{45\,\mathrm{Sinh}\left[\frac{1}{2}(c+d\,x)\right]}{2048\,d\left(2\,\mathrm{Cosh}\left[\frac{1}{2}(c+d\,x)\right]-\mathrm{Sinh}\left[\frac{1}{2}(c+d\,x)\right]\right)}+\frac{3}{512\,d\left(2\,\mathrm{Cosh}\left[\frac{1}{2}(c+d\,x)\right]+\mathrm{Sinh}\left[\frac{1}{2}(c+d\,x)\right]\right)^2}-\frac{45\,\mathrm{Sinh}\left[\frac{1}{2}(c+d\,x)\right]}{2048\,d\left(2\,\mathrm{Cosh}\left[\frac{1}{2}(c+d\,x)\right]+\mathrm{Sinh}\left[\frac{1}{2}(c+d\,x)\right]\right)}$$

## Problem 78: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{(5+3\,\mathrm{Cosh}[c+d\,x])^4}\,dx$$

Optimal (type 3, 106 leaves, 5 steps):

$$\frac{385\,x}{32768}-\frac{385\,\mathrm{ArcTanh}\left[\frac{\mathrm{Sinh}[c+d\,x]}{3+\mathrm{Cosh}[c+d\,x]}\right]}{16384\,d}-\frac{\mathrm{Sinh}[c+d\,x]}{16\,d\,(5+3\,\mathrm{Cosh}[c+d\,x])^3}-\frac{25\,\mathrm{Sinh}[c+d\,x]}{512\,d\,(5+3\,\mathrm{Cosh}[c+d\,x])^2}-\frac{311\,\mathrm{Sinh}[c+d\,x]}{8192\,d\,(5+3\,\mathrm{Cosh}[c+d\,x])}$$

Result (type 3, 296 leaves):

$$-\frac{1}{131072\,d\,(5+3\,\text{Cosh}[c+d\,x])^3}\Bigg(296450\,\text{Log}\Big[2\,\text{Cosh}\Big[\frac{1}{2}(c+d\,x)\Big]-\text{Sinh}\Big[\frac{1}{2}(c+d\,x)\Big]\Big]+10395\,\text{Cosh}[3\,(c+d\,x)]\,\text{Log}\Big[2\,\text{Cosh}\Big[\frac{1}{2}(c+d\,x)\Big]-\text{Sinh}\Big[\frac{1}{2}(c+d\,x)\Big]\Big]+$$
$$377685\,\text{Cosh}[c+d\,x]\,\Big(\text{Log}\Big[2\,\text{Cosh}\Big[\frac{1}{2}(c+d\,x)\Big]-\text{Sinh}\Big[\frac{1}{2}(c+d\,x)\Big]\Big]-\text{Log}\Big[2\,\text{Cosh}\Big[\frac{1}{2}(c+d\,x)\Big]+\text{Sinh}\Big[\frac{1}{2}(c+d\,x)\Big]\Big]\Big)+$$
$$103950\,\text{Cosh}[2\,(c+d\,x)]\,\Big(\text{Log}\Big[2\,\text{Cosh}\Big[\frac{1}{2}(c+d\,x)\Big]-\text{Sinh}\Big[\frac{1}{2}(c+d\,x)\Big]\Big]-\text{Log}\Big[2\,\text{Cosh}\Big[\frac{1}{2}(c+d\,x)\Big]+\text{Sinh}\Big[\frac{1}{2}(c+d\,x)\Big]\Big]\Big)-$$
$$296450\,\text{Log}\Big[2\,\text{Cosh}\Big[\frac{1}{2}(c+d\,x)\Big]+\text{Sinh}\Big[\frac{1}{2}(c+d\,x)\Big]\Big]-10395\,\text{Cosh}[3\,(c+d\,x)]\,\text{Log}\Big[2\,\text{Cosh}\Big[\frac{1}{2}(c+d\,x)\Big]+\text{Sinh}\Big[\frac{1}{2}(c+d\,x)\Big]\Big]+$$
$$175788\,\text{Sinh}[c+d\,x]+84240\,\text{Sinh}[2\,(c+d\,x)]+11196\,\text{Sinh}[3\,(c+d\,x)]\Bigg)$$

## Problem 197: Result more than twice size of optimal antiderivative.

$$\int\sqrt{a+b\,\text{Cosh}[x]}\,\,\text{Tanh}[x]\,dx$$

Optimal (type 3, 37 leaves, 4 steps):

$$-2\sqrt{a}\,\,\text{ArcTanh}\Big[\frac{\sqrt{a+b\,\text{Cosh}[x]}}{\sqrt{a}}\Big]+2\sqrt{a+b\,\text{Cosh}[x]}$$

Result (type 3, 75 leaves):

$$\frac{2\sqrt{a+b\,\text{Cosh}[x]}\,\left(b+a\,\text{Sech}[x]-\sqrt{a}\,\sqrt{b}\,\,\text{ArcSinh}\Big[\frac{\sqrt{a}\,\sqrt{\text{Sech}[x]}}{\sqrt{b}}\Big]\sqrt{\text{Sech}[x]}\,\sqrt{1+\frac{a\,\text{Sech}[x]}{b}}\right)}{b+a\,\text{Sech}[x]}$$

## Problem 198: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Tanh}[x]}{\sqrt{a+b\,\text{Cosh}[x]}}\,dx$$

Optimal (type 3, 24 leaves, 3 steps):

$$-\frac{2\,\text{ArcTanh}\Big[\frac{\sqrt{a+b\,\text{Cosh}[x]}}{\sqrt{a}}\Big]}{\sqrt{a}}$$

Result (type 3, 60 leaves):

$$-\frac{2\sqrt{b}\,\text{ArcSinh}\left[\frac{\sqrt{a}\,\sqrt{\text{Sech}[x]}}{\sqrt{b}}\right]\sqrt{\frac{b+a\,\text{Sech}[x]}{b}}}{\sqrt{a}\,\sqrt{a+b\,\text{Cosh}[x]}\,\sqrt{\text{Sech}[x]}}$$

## Problem 210: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x}{a+b\,\text{Cosh}[x]^2}\,dx$$

Optimal (type 4, 191 leaves, 9 steps):

$$\frac{x\,\text{Log}\left[1+\frac{b\,e^{2x}}{2a+b-2\sqrt{a}\,\sqrt{a+b}}\right]}{2\sqrt{a}\,\sqrt{a+b}}-\frac{x\,\text{Log}\left[1+\frac{b\,e^{2x}}{2a+b+2\sqrt{a}\,\sqrt{a+b}}\right]}{2\sqrt{a}\,\sqrt{a+b}}+\frac{\text{PolyLog}\left[2,-\frac{b\,e^{2x}}{2a+b-2\sqrt{a}\,\sqrt{a+b}}\right]}{4\sqrt{a}\,\sqrt{a+b}}-\frac{\text{PolyLog}\left[2,-\frac{b\,e^{2x}}{2a+b+2\sqrt{a}\,\sqrt{a+b}}\right]}{4\sqrt{a}\,\sqrt{a+b}}$$

Result (type 4, 536 leaves):

$$-\frac{1}{4\sqrt{-a\,(a+b)}}\left(4\,x\,\text{ArcTan}\left[\frac{(a+b)\,\text{Coth}[x]}{\sqrt{-a\,(a+b)}}\right]+2\,i\,\text{ArcCos}\left[-1-\frac{2\,a}{b}\right]\,\text{ArcTan}\left[\frac{a\,\text{Tanh}[x]}{\sqrt{-a\,(a+b)}}\right]+\right.$$

$$\left(\text{ArcCos}\left[-1-\frac{2\,a}{b}\right]+2\,\text{ArcTan}\left[\frac{(a+b)\,\text{Coth}[x]}{\sqrt{-a\,(a+b)}}\right]-2\,\text{ArcTan}\left[\frac{a\,\text{Tanh}[x]}{\sqrt{-a\,(a+b)}}\right]\right)\,\text{Log}\left[\frac{\sqrt{2}\,\sqrt{-a\,(a+b)}\,e^{-x}}{\sqrt{b}\,\sqrt{2\,a+b+b\,\text{Cosh}[2\,x]}}\right]+$$

$$\left(\text{ArcCos}\left[-1-\frac{2\,a}{b}\right]-2\,\text{ArcTan}\left[\frac{(a+b)\,\text{Coth}[x]}{\sqrt{-a\,(a+b)}}\right]+2\,\text{ArcTan}\left[\frac{a\,\text{Tanh}[x]}{\sqrt{-a\,(a+b)}}\right]\right)\,\text{Log}\left[\frac{\sqrt{2}\,\sqrt{-a\,(a+b)}\,e^{x}}{\sqrt{b}\,\sqrt{2\,a+b+b\,\text{Cosh}[2\,x]}}\right]-$$

$$\left(\text{ArcCos}\left[-1-\frac{2\,a}{b}\right]-2\,\text{ArcTan}\left[\frac{a\,\text{Tanh}[x]}{\sqrt{-a\,(a+b)}}\right]\right)\,\text{Log}\left[\frac{2\,(a+b)\,\left(a+i\,\sqrt{-a\,(a+b)}\right)\,(-1+\text{Tanh}[x])}{b\,\left(a+b+i\,\sqrt{-a\,(a+b)}\,\text{Tanh}[x]\right)}\right]-$$

$$\left(\text{ArcCos}\left[-1-\frac{2\,a}{b}\right]+2\,\text{ArcTan}\left[\frac{a\,\text{Tanh}[x]}{\sqrt{-a\,(a+b)}}\right]\right)\,\text{Log}\left[\frac{2\,i\,(a+b)\,\left(i\,a+\sqrt{-a\,(a+b)}\right)\,(1+\text{Tanh}[x])}{b\,\left(a+b+i\,\sqrt{-a\,(a+b)}\,\text{Tanh}[x]\right)}\right]+$$

$$i\left(\text{PolyLog}\left[2,\frac{\left(2\,a+b-2\,i\,\sqrt{-a\,(a+b)}\right)\,\left(a+b-i\,\sqrt{-a\,(a+b)}\,\text{Tanh}[x]\right)}{b\,\left(a+b+i\,\sqrt{-a\,(a+b)}\,\text{Tanh}[x]\right)}\right]-\right.$$

$$\left.\left.\text{PolyLog}\left[2,\frac{\left(2\,a+b+2\,i\,\sqrt{-a\,(a+b)}\right)\,\left(a+b-i\,\sqrt{-a\,(a+b)}\,\text{Tanh}[x]\right)}{b\,\left(a+b+i\,\sqrt{-a\,(a+b)}\,\text{Tanh}[x]\right)}\right]\right)\right)$$

## Problem 224: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\,\text{Sinh}[c+d\,x]}{a+b\,\text{Cosh}[c+d\,x]}\,dx$$

Optimal (type 4, 161 leaves, 7 steps):

$$-\frac{x^2}{2\,b}+\frac{x\,\text{Log}\left[1+\frac{b\,e^{c+d\,x}}{a-\sqrt{a^2-b^2}}\right]}{b\,d}+\frac{x\,\text{Log}\left[1+\frac{b\,e^{c+d\,x}}{a+\sqrt{a^2-b^2}}\right]}{b\,d}+\frac{\text{PolyLog}\left[2,-\frac{b\,e^{c+d\,x}}{a-\sqrt{a^2-b^2}}\right]}{b\,d^2}+\frac{\text{PolyLog}\left[2,-\frac{b\,e^{c+d\,x}}{a+\sqrt{a^2-b^2}}\right]}{b\,d^2}$$

Result (type 4, 279 leaves):

$$\frac{1}{b\,d^2}\left(\frac{1}{2}(c+d\,x)^2+4\,i\,\text{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right]\text{ArcTanh}\left[\frac{(a-b)\,\text{Tanh}\left[\frac{1}{2}(c+d\,x)\right]}{\sqrt{a^2-b^2}}\right]+\right.$$

$$\left(c+d\,x-2\,i\,\text{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right]\right)\text{Log}\left[1+\frac{\left(a-\sqrt{a^2-b^2}\right)e^{-c-d\,x}}{b}\right]+\left(c+d\,x+2\,i\,\text{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right]\right)\text{Log}\left[1+\frac{\left(a+\sqrt{a^2-b^2}\right)e^{-c-d\,x}}{b}\right]-$$

$$\left.c\,\text{Log}\left[1+\frac{b\,\text{Cosh}[c+d\,x]}{a}\right]-\text{PolyLog}\left[2,\frac{\left(-a+\sqrt{a^2-b^2}\right)e^{-c-d\,x}}{b}\right]-\text{PolyLog}\left[2,-\frac{\left(a+\sqrt{a^2-b^2}\right)e^{-c-d\,x}}{b}\right]\right)$$

## Problem 232: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{Sinh}[c+d\,x]^2}{x\,(a+b\,\text{Cosh}[c+d\,x])}\,dx$$

Optimal (type 9, 26 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\text{Sinh}[c+d\,x]^2}{x\,(a+b\,\text{Cosh}[c+d\,x])},x\right]$$

Result (type 1, 1 leaves):

???

## Problem 236: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x\,\text{Sinh}[c+d\,x]^3}{a+b\,\text{Cosh}[c+d\,x]}\,dx$$

Optimal (type 4, 288 leaves, 13 steps):

$$\frac{x}{4 b d}-\frac{\left(a^2-b^2\right) x^2}{2 b^3}-\frac{a x \text{Cosh}[c+d x]}{b^2 d}+\frac{\left(a^2-b^2\right) x \text{Log}\left[1+\frac{b e^{c+d x}}{a-\sqrt{a^2-b^2}}\right]}{b^3 d}+\frac{\left(a^2-b^2\right) x \text{Log}\left[1+\frac{b e^{c+d x}}{a+\sqrt{a^2-b^2}}\right]}{b^3 d}+$$

$$\frac{\left(a^2-b^2\right) \text{PolyLog}\left[2,-\frac{b e^{c+d x}}{a-\sqrt{a^2-b^2}}\right]}{b^3 d^2}+\frac{\left(a^2-b^2\right) \text{PolyLog}\left[2,-\frac{b e^{c+d x}}{a+\sqrt{a^2-b^2}}\right]}{b^3 d^2}+\frac{a \text{Sinh}[c+d x]}{b^2 d^2}-\frac{\text{Cosh}[c+d x] \text{Sinh}[c+d x]}{4 b d^2}+\frac{x \text{Sinh}[c+d x]^2}{2 b d}$$

Result (type 4, 621 leaves):

$$\frac{1}{8 b^3 d^2}\left(-8 a b d x \text{Cosh}[c+d x]+2 b^2 d x \text{Cosh}[2 (c+d x)]-8 a^2 c \text{Log}\left[1+\frac{b \text{Cosh}[c+d x]}{a}\right]+8 b^2 c \text{Log}\left[1+\frac{b \text{Cosh}[c+d x]}{a}\right]+\right.$$

$$8 a^2 \left(\frac{1}{2} (c+d x)^2+4 i \text{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \text{ArcTanh}\left[\frac{(a-b) \text{Tanh}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^2-b^2}}\right]+\left(c+d x-2 i \text{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right]\right) \text{Log}\left[1+\frac{\left(a-\sqrt{a^2-b^2}\right) e^{-c-d x}}{b}\right]+\right.$$

$$\left.\left(c+d x+2 i \text{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right]\right) \text{Log}\left[1+\frac{\left(a+\sqrt{a^2-b^2}\right) e^{-c-d x}}{b}\right]-\text{PolyLog}\left[2,\frac{\left(-a+\sqrt{a^2-b^2}\right) e^{-c-d x}}{b}\right]-\text{PolyLog}\left[2,-\frac{\left(a+\sqrt{a^2-b^2}\right) e^{-c-d x}}{b}\right]\right)-$$

$$8 b^2 \left(\frac{1}{2} (c+d x)^2+4 i \text{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \text{ArcTanh}\left[\frac{(a-b) \text{Tanh}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^2-b^2}}\right]+\left(c+d x-2 i \text{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right]\right) \text{Log}\left[1+\frac{\left(a-\sqrt{a^2-b^2}\right) e^{-c-d x}}{b}\right]+\right.$$

$$\left(c+d x+2 i \text{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right]\right) \text{Log}\left[1+\frac{\left(a+\sqrt{a^2-b^2}\right) e^{-c-d x}}{b}\right]-\text{PolyLog}\left[2,\frac{\left(-a+\sqrt{a^2-b^2}\right) e^{-c-d x}}{b}\right]-$$

$$\left.\left.\text{PolyLog}\left[2,-\frac{\left(a+\sqrt{a^2-b^2}\right) e^{-c-d x}}{b}\right]\right)+8 a b \text{Sinh}[c+d x]-b^2 \text{Sinh}[2 (c+d x)]\right)$$

## Problem 238: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{Sinh}[c+d x]^3}{x\,(a+b\,\text{Cosh}[c+d x])}\,dx$$

Optimal (type 9, 26 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\text{Sinh}[c+d x]^3}{x\,(a+b\,\text{Cosh}[c+d x])},\,x\right]$$

Result (type 1, 1 leaves):

???

## Problem 247: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cosh}[a+b\,\text{Log}[c\,x^n]]}{x}\,dx$$

Optimal (type 3, 18 leaves, 2 steps):

$$\frac{\text{Sinh}[a+b\,\text{Log}[c\,x^n]]}{b\,n}$$

Result (type 3, 37 leaves):

$$\frac{\text{Cosh}[b\,\text{Log}[c\,x^n]]\,\text{Sinh}[a]}{b\,n}+\frac{\text{Cosh}[a]\,\text{Sinh}[b\,\text{Log}[c\,x^n]]}{b\,n}$$

## Problem 262: Result more than twice size of optimal antiderivative.

$$\int \text{Cosh}\left[\frac{a+b x}{c+d x}\right]dx$$

Optimal (type 4, 101 leaves, 5 steps):

$$\frac{(c+d x)\,\text{Cosh}\left[\frac{a+b x}{c+d x}\right]}{d}+\frac{(b c-a d)\,\text{CoshIntegral}\left[\frac{b c-a d}{d\,(c+d x)}\right]\text{Sinh}\left[\frac{b}{d}\right]}{d^2}-\frac{(b c-a d)\,\text{Cosh}\left[\frac{b}{d}\right]\text{SinhIntegral}\left[\frac{b c-a d}{d\,(c+d x)}\right]}{d^2}$$

Result (type 4, 373 leaves):

$$\frac{1}{2 d^2}\left(2 c d \operatorname{Cosh}\left[\frac{a+b x}{c+d x}\right]+2 d^2 x \operatorname{Cosh}\left[\frac{a+b x}{c+d x}\right]+(b c-a d) \operatorname{CoshIntegral}\left[\frac{b c-a d}{c d+d^2 x}\right]\left(-\operatorname{Cosh}\left[\frac{b}{d}\right]+\operatorname{Sinh}\left[\frac{b}{d}\right]\right)+\right.$$
$$(b c-a d) \operatorname{CoshIntegral}\left[\frac{-b c+a d}{d(c+d x)}\right]\left(\operatorname{Cosh}\left[\frac{b}{d}\right]+\operatorname{Sinh}\left[\frac{b}{d}\right]\right)+b c \operatorname{Cosh}\left[\frac{b}{d}\right] \operatorname{SinhIntegral}\left[\frac{-b c+a d}{d(c+d x)}\right]-a d \operatorname{Cosh}\left[\frac{b}{d}\right] \operatorname{SinhIntegral}\left[\frac{-b c+a d}{d(c+d x)}\right]+$$
$$b c \operatorname{Sinh}\left[\frac{b}{d}\right] \operatorname{SinhIntegral}\left[\frac{-b c+a d}{d(c+d x)}\right]-a d \operatorname{Sinh}\left[\frac{b}{d}\right] \operatorname{SinhIntegral}\left[\frac{-b c+a d}{d(c+d x)}\right]-b c \operatorname{Cosh}\left[\frac{b}{d}\right] \operatorname{SinhIntegral}\left[\frac{b c-a d}{c d+d^2 x}\right]+$$
$$\left.a d \operatorname{Cosh}\left[\frac{b}{d}\right] \operatorname{SinhIntegral}\left[\frac{b c-a d}{c d+d^2 x}\right]+b c \operatorname{Sinh}\left[\frac{b}{d}\right] \operatorname{SinhIntegral}\left[\frac{b c-a d}{c d+d^2 x}\right]-a d \operatorname{Sinh}\left[\frac{b}{d}\right] \operatorname{SinhIntegral}\left[\frac{b c-a d}{c d+d^2 x}\right]\right)$$

## Problem 275: Result is not expressed in closed-form.

$$\int e^x \operatorname{Sech}[2 x]\, dx$$

Optimal (type 3, 92 leaves, 11 steps):

$$-\frac{\operatorname{ArcTan}\left[1-\sqrt{2}\, e^x\right]}{\sqrt{2}}+\frac{\operatorname{ArcTan}\left[1+\sqrt{2}\, e^x\right]}{\sqrt{2}}+\frac{\operatorname{Log}\left[1-\sqrt{2}\, e^x+e^{2 x}\right]}{2 \sqrt{2}}-\frac{\operatorname{Log}\left[1+\sqrt{2}\, e^x+e^{2 x}\right]}{2 \sqrt{2}}$$

Result (type 7, 31 leaves):

$$-\frac{1}{2} \operatorname{RootSum}\left[1+\#1^4\, \&, \frac{x-\operatorname{Log}\left[e^x-\#1\right]}{\#1}\, \&\right]$$

## Problem 276: Result is not expressed in closed-form.

$$\int e^x \operatorname{Sech}[2 x]^2\, dx$$

Optimal (type 3, 111 leaves, 12 steps):

$$-\frac{e^x}{1+e^{4 x}}-\frac{\operatorname{ArcTan}\left[1-\sqrt{2}\, e^x\right]}{2 \sqrt{2}}+\frac{\operatorname{ArcTan}\left[1+\sqrt{2}\, e^x\right]}{2 \sqrt{2}}-\frac{\operatorname{Log}\left[1-\sqrt{2}\, e^x+e^{2 x}\right]}{4 \sqrt{2}}+\frac{\operatorname{Log}\left[1+\sqrt{2}\, e^x+e^{2 x}\right]}{4 \sqrt{2}}$$

Result (type 7, 46 leaves):

$$-\frac{e^x}{1+e^{4 x}}-\frac{1}{4} \operatorname{RootSum}\left[1+\#1^4\, \&, \frac{x-\operatorname{Log}\left[e^x-\#1\right]}{\#1^3}\, \&\right]$$

## Problem 279: Result is not expressed in closed-form.

$$\int e^x \operatorname{Sech}[3 x]\, dx$$

Optimal (type 3, 55 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-2 e^{2x}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{1}{3}\text{Log}\left[1+e^{2x}\right]+\frac{1}{6}\text{Log}\left[1-e^{2x}+e^{4x}\right]$$

Result (type 7, 55 leaves):

$$\frac{2x}{3}-\frac{1}{3}\text{Log}\left[1+e^{2x}\right]-\frac{1}{3}\text{RootSum}\left[1-\#1^2+\#1^4\,\&,\ \frac{x-\text{Log}\left[e^x-\#1\right]}{\#1^2}\,\&\right]$$

## Problem 280: Result is not expressed in closed-form.

$$\int e^x\,\text{Sech}[3x]^2\,dx$$

Optimal (type 3, 110 leaves, 13 steps):

$$-\frac{2 e^x}{3\left(1+e^{6x}\right)}+\frac{2\,\text{ArcTan}\left[e^x\right]}{9}-\frac{1}{9}\text{ArcTan}\left[\sqrt{3}-2 e^x\right]+\frac{1}{9}\text{ArcTan}\left[\sqrt{3}+2 e^x\right]-\frac{\text{Log}\left[1-\sqrt{3}\,e^x+e^{2x}\right]}{6\sqrt{3}}+\frac{\text{Log}\left[1+\sqrt{3}\,e^x+e^{2x}\right]}{6\sqrt{3}}$$

Result (type 7, 90 leaves):

$$\frac{1}{9}\left(-\frac{6 e^x}{1+e^{6x}}+2\,\text{ArcTan}\left[e^x\right]+\text{RootSum}\left[1-\#1^2+\#1^4\,\&,\ \frac{-2x+2\,\text{Log}\left[e^x-\#1\right]+x\,\#1^2-\text{Log}\left[e^x-\#1\right]\#1^2}{-\#1+2\,\#1^3}\,\&\right]\right)$$

## Problem 283: Result is not expressed in closed-form.

$$\int e^x\,\text{Sech}[4x]\,dx$$

Optimal (type 3, 371 leaves, 21 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right]}{2\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right]}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}+2e^x}{\sqrt{2+\sqrt{2}}}\right]}{2\sqrt{2\left(2+\sqrt{2}\right)}}+\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}+2e^x}{\sqrt{2-\sqrt{2}}}\right]}{2\sqrt{2\left(2-\sqrt{2}\right)}}-$$

$$\frac{\text{Log}\left[1-\sqrt{2-\sqrt{2}}\,e^x+e^{2x}\right]}{4\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\text{Log}\left[1+\sqrt{2-\sqrt{2}}\,e^x+e^{2x}\right]}{4\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\text{Log}\left[1-\sqrt{2+\sqrt{2}}\,e^x+e^{2x}\right]}{4\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\text{Log}\left[1+\sqrt{2+\sqrt{2}}\,e^x+e^{2x}\right]}{4\sqrt{2\left(2+\sqrt{2}\right)}}$$

Result (type 7, 31 leaves):

$$-\frac{1}{4}\,\text{RootSum}\left[1+\#1^8\,\&,\ \frac{x-\text{Log}\left[e^x-\#1\right]}{\#1^3}\,\&\right]$$

## Problem 284: Result is not expressed in closed-form.

$$\int e^x\,\text{Sech}[4x]^2\,dx$$

Optimal (type 3, 379 leaves, 22 steps):

$$-\frac{e^x}{2\left(1+e^{8x}\right)}-\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right]}{8\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right]}{8\sqrt{2\left(2+\sqrt{2}\right)}}+\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}+2e^x}{\sqrt{2+\sqrt{2}}}\right]}{8\sqrt{2\left(2-\sqrt{2}\right)}}+$$

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}+2e^x}{\sqrt{2-\sqrt{2}}}\right]}{8\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{1}{32}\sqrt{2-\sqrt{2}}\,\text{Log}\left[1-\sqrt{2-\sqrt{2}}\,e^x+e^{2x}\right]+\frac{1}{32}\sqrt{2-\sqrt{2}}\,\text{Log}\left[1+\sqrt{2-\sqrt{2}}\,e^x+e^{2x}\right]-$$

$$\frac{1}{32}\sqrt{2+\sqrt{2}}\,\text{Log}\left[1-\sqrt{2+\sqrt{2}}\,e^x+e^{2x}\right]+\frac{1}{32}\sqrt{2+\sqrt{2}}\,\text{Log}\left[1+\sqrt{2+\sqrt{2}}\,e^x+e^{2x}\right]$$

Result (type 7, 48 leaves):

$$-\frac{e^x}{2\left(1+e^{8x}\right)}-\frac{1}{16}\,\text{RootSum}\left[1+\#1^8\,\&,\ \frac{x-\text{Log}\left[e^x-\#1\right]}{\#1^7}\,\&\right]$$

## Problem 288: Unable to integrate problem.

$$\int F^{c\,(a+b\,x)}\,\text{Sech}[d+e\,x]\,dx$$

Optimal (type 5, 68 leaves, 1 step):

$$\frac{2\,e^{d+e\,x}\,F^{c\,(a+b\,x)}\,\text{Hypergeometric2F1}\left[1,\ \frac{e+b\,c\,\text{Log}[F]}{2\,e},\ \frac{1}{2}\left(3+\frac{b\,c\,\text{Log}[F]}{e}\right),\ -e^{2\,(d+e\,x)}\right]}{e+b\,c\,\text{Log}[F]}$$

Result (type 8, 18 leaves):

$$\int F^{c\,(a+b\,x)}\,\text{Sech}[d+e\,x]\,dx$$

## Problem 290: Unable to integrate problem.

$$\int F^{c\,(a+b\,x)}\,\mathrm{Sech}[d+e\,x]^3\,dx$$

Optimal (type 5, 124 leaves, 2 steps):

$$\frac{e^{d+e\,x}\,F^{c\,(a+b\,x)}\,\mathrm{Hypergeometric2F1}\left[1,\frac{e+b\,c\,\mathrm{Log}[F]}{2\,e},\frac{1}{2}\left(3+\frac{b\,c\,\mathrm{Log}[F]}{e}\right),-e^{2\,(d+e\,x)}\right]\,(e-b\,c\,\mathrm{Log}[F])}{e^2}+\frac{b\,c\,F^{c\,(a+b\,x)}\,\mathrm{Log}[F]\,\mathrm{Sech}[d+e\,x]}{2\,e^2}+\frac{F^{c\,(a+b\,x)}\,\mathrm{Sech}[d+e\,x]\,\mathrm{Tanh}[d+e\,x]}{2\,e}$$

Result (type 8, 20 leaves):

$$\int F^{c\,(a+b\,x)}\,\mathrm{Sech}[d+e\,x]^3\,dx$$

## Problem 319: Result more than twice size of optimal antiderivative.

$$\int f^{a+c\,x^2}\,\mathrm{Cosh}\left[d+e\,x+f\,x^2\right]^3\,dx$$

Optimal (type 4, 300 leaves, 14 steps):

$$\frac{3\,e^{-d+\frac{e^2}{4\,f-4\,c\,\mathrm{Log}[f]}}\,f^a\,\sqrt{\pi}\,\mathrm{Erf}\left[\frac{e+2\,x\,(f-c\,\mathrm{Log}[f])}{2\,\sqrt{f-c\,\mathrm{Log}[f]}}\right]}{16\,\sqrt{f-c\,\mathrm{Log}[f]}}+\frac{e^{-3\,d+\frac{9\,e^2}{12\,f-4\,c\,\mathrm{Log}[f]}}\,f^a\,\sqrt{\pi}\,\mathrm{Erf}\left[\frac{3\,e+2\,x\,(3\,f-c\,\mathrm{Log}[f])}{2\,\sqrt{3\,f-c\,\mathrm{Log}[f]}}\right]}{16\,\sqrt{3\,f-c\,\mathrm{Log}[f]}}+$$

$$\frac{3\,e^{d-\frac{e^2}{4\,(f+c\,\mathrm{Log}[f])}}\,f^a\,\sqrt{\pi}\,\mathrm{Erfi}\left[\frac{e+2\,x\,(f+c\,\mathrm{Log}[f])}{2\,\sqrt{f+c\,\mathrm{Log}[f]}}\right]}{16\,\sqrt{f+c\,\mathrm{Log}[f]}}+\frac{e^{3\,d-\frac{9\,e^2}{4\,(3\,f+c\,\mathrm{Log}[f])}}\,f^a\,\sqrt{\pi}\,\mathrm{Erfi}\left[\frac{3\,e+2\,x\,(3\,f+c\,\mathrm{Log}[f])}{2\,\sqrt{3\,f+c\,\mathrm{Log}[f]}}\right]}{16\,\sqrt{3\,f+c\,\mathrm{Log}[f]}}$$

Result (type 4, 2303 leaves):

$$\frac{1}{16\,(f-c\,\mathrm{Log}[f])\,(3\,f-c\,\mathrm{Log}[f])\,(f+c\,\mathrm{Log}[f])\,(3\,f+c\,\mathrm{Log}[f])}$$

$$f^a\,\sqrt{\pi}\,\Bigg(27\,e^{\frac{e^2}{4\,(f-c\,\mathrm{Log}[f])}}\,f^3\,\mathrm{Cosh}[d]\,\mathrm{Erf}\left[\frac{e+2\,f\,x-2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{f-c\,\mathrm{Log}[f]}}\right]\,\sqrt{f-c\,\mathrm{Log}[f]}+27\,c\,e^{\frac{e^2}{4\,(f-c\,\mathrm{Log}[f])}}\,f^2\,\mathrm{Cosh}[d]$$

$$\mathrm{Erf}\left[\frac{e+2\,f\,x-2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{f-c\,\mathrm{Log}[f]}}\right]\,\mathrm{Log}[f]\,\sqrt{f-c\,\mathrm{Log}[f]}-3\,c^2\,e^{\frac{e^2}{4\,(f-c\,\mathrm{Log}[f])}}\,f\,\mathrm{Cosh}[d]\,\mathrm{Erf}\left[\frac{e+2\,f\,x-2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{f-c\,\mathrm{Log}[f]}}\right]\,\mathrm{Log}[f]^2\,\sqrt{f-c\,\mathrm{Log}[f]}-$$

$$3\,c^3\,e^{\frac{e^2}{4\,(f-c\,\mathrm{Log}[f])}}\,\mathrm{Cosh}[d]\,\mathrm{Erf}\left[\frac{e+2\,f\,x-2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{f-c\,\mathrm{Log}[f]}}\right]\,\mathrm{Log}[f]^3\,\sqrt{f-c\,\mathrm{Log}[f]}+3\,e^{\frac{9\,e^2}{4\,(3\,f-c\,\mathrm{Log}[f])}}\,f^3\,\mathrm{Cosh}[3\,d]\,\mathrm{Erf}\left[\frac{3\,e+6\,f\,x-2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{3\,f-c\,\mathrm{Log}[f]}}\right]$$

$$\sqrt{3\,f-c\,\text{Log}[f]}\;+c\,e^{\frac{9\,e^2}{4\,(3\,f-c\,\text{Log}[f])}}\,f^2\,\text{Cosh}[3\,d]\,\text{Erf}\Big[\frac{3\,e+6\,f\,x-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f-c\,\text{Log}[f]}}\Big]\,\text{Log}[f]\,\sqrt{3\,f-c\,\text{Log}[f]}\;-$$

$$3\,c^2\,e^{\frac{9\,e^2}{4\,(3\,f-c\,\text{Log}[f])}}\,f\,\text{Cosh}[3\,d]\,\text{Erf}\Big[\frac{3\,e+6\,f\,x-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f-c\,\text{Log}[f]}}\Big]\,\text{Log}[f]^2\,\sqrt{3\,f-c\,\text{Log}[f]}\;-c^3\,e^{\frac{9\,e^2}{4\,(3\,f-c\,\text{Log}[f])}}\,\text{Cosh}[3\,d]$$

$$\text{Erf}\Big[\frac{3\,e+6\,f\,x-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f-c\,\text{Log}[f]}}\Big]\,\text{Log}[f]^3\,\sqrt{3\,f-c\,\text{Log}[f]}\;+27\,e^{-\frac{e^2}{4\,(f+c\,\text{Log}[f])}}\,f^3\,\text{Cosh}[d]\,\text{Erfi}\Big[\frac{e+2\,f\,x+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f+c\,\text{Log}[f]}}\Big]\,\sqrt{f+c\,\text{Log}[f]}\;-$$

$$27\,c\,e^{-\frac{e^2}{4\,(f+c\,\text{Log}[f])}}\,f^2\,\text{Cosh}[d]\,\text{Erfi}\Big[\frac{e+2\,f\,x+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f+c\,\text{Log}[f]}}\Big]\,\text{Log}[f]\,\sqrt{f+c\,\text{Log}[f]}\;-3\,c^2\,e^{-\frac{e^2}{4\,(f+c\,\text{Log}[f])}}\,f\,\text{Cosh}[d]\,\text{Erfi}\Big[\frac{e+2\,f\,x+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f+c\,\text{Log}[f]}}\Big]$$

$$\text{Log}[f]^2\,\sqrt{f+c\,\text{Log}[f]}\;+3\,c^3\,e^{-\frac{e^2}{4\,(f+c\,\text{Log}[f])}}\,\text{Cosh}[d]\,\text{Erfi}\Big[\frac{e+2\,f\,x+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f+c\,\text{Log}[f]}}\Big]\,\text{Log}[f]^3\,\sqrt{f+c\,\text{Log}[f]}\;+$$

$$3\,e^{-\frac{9\,e^2}{4\,(3\,f+c\,\text{Log}[f])}}\,f^3\,\text{Cosh}[3\,d]\,\text{Erfi}\Big[\frac{3\,e+6\,f\,x+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f+c\,\text{Log}[f]}}\Big]\,\sqrt{3\,f+c\,\text{Log}[f]}\;-c\,e^{-\frac{9\,e^2}{4\,(3\,f+c\,\text{Log}[f])}}\,f^2\,\text{Cosh}[3\,d]\,\text{Erfi}\Big[\frac{3\,e+6\,f\,x+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f+c\,\text{Log}[f]}}\Big]$$

$$\text{Log}[f]\,\sqrt{3\,f+c\,\text{Log}[f]}\;-3\,c^2\,e^{-\frac{9\,e^2}{4\,(3\,f+c\,\text{Log}[f])}}\,f\,\text{Cosh}[3\,d]\,\text{Erfi}\Big[\frac{3\,e+6\,f\,x+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f+c\,\text{Log}[f]}}\Big]\,\text{Log}[f]^2\,\sqrt{3\,f+c\,\text{Log}[f]}\;+$$

$$c^3\,e^{-\frac{9\,e^2}{4\,(3\,f+c\,\text{Log}[f])}}\,\text{Cosh}[3\,d]\,\text{Erfi}\Big[\frac{3\,e+6\,f\,x+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f+c\,\text{Log}[f]}}\Big]\,\text{Log}[f]^3\,\sqrt{3\,f+c\,\text{Log}[f]}\;-27\,e^{\frac{e^2}{4\,(f-c\,\text{Log}[f])}}\,f^3\,\text{Erf}\Big[\frac{e+2\,f\,x-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f-c\,\text{Log}[f]}}\Big]$$

$$\sqrt{f-c\,\text{Log}[f]}\;\text{Sinh}[d]-27\,c\,e^{\frac{e^2}{4\,(f-c\,\text{Log}[f])}}\,f^2\,\text{Erf}\Big[\frac{e+2\,f\,x-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f-c\,\text{Log}[f]}}\Big]\,\text{Log}[f]\,\sqrt{f-c\,\text{Log}[f]}\;\text{Sinh}[d]+$$

$$3\,c^2\,e^{\frac{e^2}{4\,(f-c\,\text{Log}[f])}}\,f\,\text{Erf}\Big[\frac{e+2\,f\,x-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f-c\,\text{Log}[f]}}\Big]\,\text{Log}[f]^2\,\sqrt{f-c\,\text{Log}[f]}\;\text{Sinh}[d]+3\,c^3\,e^{\frac{e^2}{4\,(f-c\,\text{Log}[f])}}\,\text{Erf}\Big[\frac{e+2\,f\,x-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f-c\,\text{Log}[f]}}\Big]$$

$$\text{Log}[f]^3\,\sqrt{f-c\,\text{Log}[f]}\;\text{Sinh}[d]+27\,e^{-\frac{e^2}{4\,(f+c\,\text{Log}[f])}}\,f^3\,\text{Erfi}\Big[\frac{e+2\,f\,x+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f+c\,\text{Log}[f]}}\Big]\,\sqrt{f+c\,\text{Log}[f]}\;\text{Sinh}[d]-$$

$$27\,c\,e^{-\frac{e^2}{4\,(f+c\,\text{Log}[f])}}\,f^2\,\text{Erfi}\Big[\frac{e+2\,f\,x+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f+c\,\text{Log}[f]}}\Big]\,\text{Log}[f]\,\sqrt{f+c\,\text{Log}[f]}\;\text{Sinh}[d]-3\,c^2\,e^{-\frac{e^2}{4\,(f+c\,\text{Log}[f])}}\,f\,\text{Erfi}\Big[\frac{e+2\,f\,x+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f+c\,\text{Log}[f]}}\Big]$$

$$\text{Log}[f]^2\,\sqrt{f+c\,\text{Log}[f]}\;\text{Sinh}[d]+3\,c^3\,e^{-\frac{e^2}{4\,(f+c\,\text{Log}[f])}}\,\text{Erfi}\Big[\frac{e+2\,f\,x+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f+c\,\text{Log}[f]}}\Big]\,\text{Log}[f]^3\,\sqrt{f+c\,\text{Log}[f]}\;\text{Sinh}[d]-$$

$$3\,e^{\frac{9\,e^2}{4\,(3\,f-c\,\text{Log}[f])}}\,f^3\,\text{Erf}\Big[\frac{3\,e+6\,f\,x-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f-c\,\text{Log}[f]}}\Big]\,\sqrt{3\,f-c\,\text{Log}[f]}\;\text{Sinh}[3\,d]-c\,e^{\frac{9\,e^2}{4\,(3\,f-c\,\text{Log}[f])}}\,f^2\,\text{Erf}\Big[\frac{3\,e+6\,f\,x-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f-c\,\text{Log}[f]}}\Big]$$

$$\text{Log}[f]\,\sqrt{3\,f-c\,\text{Log}[f]}\;\text{Sinh}[3\,d]+3\,c^2\,e^{\frac{9\,e^2}{4\,(3\,f-c\,\text{Log}[f])}}\,f\,\text{Erf}\Big[\frac{3\,e+6\,f\,x-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f-c\,\text{Log}[f]}}\Big]\,\text{Log}[f]^2\,\sqrt{3\,f-c\,\text{Log}[f]}\;\text{Sinh}[3\,d]+$$

$$c^3\, e^{\frac{9 e^2}{4 (3 f-c \operatorname{Log}[f])}} \operatorname{Erf}\left[\frac{3 e+6 f x-2 c x \operatorname{Log}[f]}{2 \sqrt{3 f-c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{3 f-c \operatorname{Log}[f]}\, \operatorname{Sinh}[3 d]+3\, e^{-\frac{9 e^2}{4 (3 f+c \operatorname{Log}[f])}} f^3 \operatorname{Erfi}\left[\frac{3 e+6 f x+2 c x \operatorname{Log}[f]}{2 \sqrt{3 f+c \operatorname{Log}[f]}}\right]$$

$$\sqrt{3 f+c \operatorname{Log}[f]}\, \operatorname{Sinh}[3 d]-c\, e^{-\frac{9 e^2}{4 (3 f+c \operatorname{Log}[f])}} f^2 \operatorname{Erfi}\left[\frac{3 e+6 f x+2 c x \operatorname{Log}[f]}{2 \sqrt{3 f+c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{3 f+c \operatorname{Log}[f]}\, \operatorname{Sinh}[3 d]-$$

$$3 c^2\, e^{-\frac{9 e^2}{4 (3 f+c \operatorname{Log}[f])}} f \operatorname{Erfi}\left[\frac{3 e+6 f x+2 c x \operatorname{Log}[f]}{2 \sqrt{3 f+c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{3 f+c \operatorname{Log}[f]}\, \operatorname{Sinh}[3 d]+$$

$$\left.c^3\, e^{-\frac{9 e^2}{4 (3 f+c \operatorname{Log}[f])}} \operatorname{Erfi}\left[\frac{3 e+6 f x+2 c x \operatorname{Log}[f]}{2 \sqrt{3 f+c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{3 f+c \operatorname{Log}[f]}\, \operatorname{Sinh}[3 d]\right)$$

## Problem 325: Result more than twice size of optimal antiderivative.

$$\int f^{a+b x+c x^2} \operatorname{Cosh}\left[d+f x^2\right]^3 dx$$

Optimal (type 4, 323 leaves, 14 steps):

$$-\frac{3\, e^{-d+\frac{b^2 \operatorname{Log}[f]^2}{4 f-4 c \operatorname{Log}[f]}} f^a \sqrt{\pi}\, \operatorname{Erf}\left[\frac{b \operatorname{Log}[f]-2 x (f-c \operatorname{Log}[f])}{2 \sqrt{f-c \operatorname{Log}[f]}}\right]}{16 \sqrt{f-c \operatorname{Log}[f]}}-\frac{e^{-3 d+\frac{b^2 \operatorname{Log}[f]^2}{12 f-4 c \operatorname{Log}[f]}} f^a \sqrt{\pi}\, \operatorname{Erf}\left[\frac{b \operatorname{Log}[f]-2 x (3 f-c \operatorname{Log}[f])}{2 \sqrt{3 f-c \operatorname{Log}[f]}}\right]}{16 \sqrt{3 f-c \operatorname{Log}[f]}}+$$

$$\frac{3\, e^{d-\frac{b^2 \operatorname{Log}[f]^2}{4 (f+c \operatorname{Log}[f])}} f^a \sqrt{\pi}\, \operatorname{Erfi}\left[\frac{b \operatorname{Log}[f]+2 x (f+c \operatorname{Log}[f])}{2 \sqrt{f+c \operatorname{Log}[f]}}\right]}{16 \sqrt{f+c \operatorname{Log}[f]}}+\frac{e^{3 d-\frac{b^2 \operatorname{Log}[f]^2}{4 (3 f+c \operatorname{Log}[f])}} f^a \sqrt{\pi}\, \operatorname{Erfi}\left[\frac{b \operatorname{Log}[f]+2 x (3 f+c \operatorname{Log}[f])}{2 \sqrt{3 f+c \operatorname{Log}[f]}}\right]}{16 \sqrt{3 f+c \operatorname{Log}[f]}}$$

Result (type 4, 2511 leaves):

$$\frac{1}{16 (f-c \operatorname{Log}[f]) (3 f-c \operatorname{Log}[f]) (f+c \operatorname{Log}[f]) (3 f+c \operatorname{Log}[f])}$$

$$f^a \sqrt{\pi} \left(27\, e^{\frac{b^2 \operatorname{Log}[f]^2}{4 (f-c \operatorname{Log}[f])}} f^3 \operatorname{Cosh}[d] \operatorname{Erf}\left[\frac{2 f x-b \operatorname{Log}[f]-2 c x \operatorname{Log}[f]}{2 \sqrt{f-c \operatorname{Log}[f]}}\right] \sqrt{f-c \operatorname{Log}[f]}+27 c\, e^{\frac{b^2 \operatorname{Log}[f]^2}{4 (f-c \operatorname{Log}[f])}} f^2 \operatorname{Cosh}[d] \operatorname{Erf}\left[\frac{2 f x-b \operatorname{Log}[f]-2 c x \operatorname{Log}[f]}{2 \sqrt{f-c \operatorname{Log}[f]}}\right]\right.$$

$$\operatorname{Log}[f] \sqrt{f-c \operatorname{Log}[f]}-3 c^2\, e^{\frac{b^2 \operatorname{Log}[f]^2}{4 (f-c \operatorname{Log}[f])}} f \operatorname{Cosh}[d] \operatorname{Erf}\left[\frac{2 f x-b \operatorname{Log}[f]-2 c x \operatorname{Log}[f]}{2 \sqrt{f-c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{f-c \operatorname{Log}[f]}-$$

$$3 c^3\, e^{\frac{b^2 \operatorname{Log}[f]^2}{4 (f-c \operatorname{Log}[f])}} \operatorname{Cosh}[d] \operatorname{Erf}\left[\frac{2 f x-b \operatorname{Log}[f]-2 c x \operatorname{Log}[f]}{2 \sqrt{f-c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{f-c \operatorname{Log}[f]}+3\, e^{\frac{b^2 \operatorname{Log}[f]^2}{4 (3 f-c \operatorname{Log}[f])}} f^3 \operatorname{Cosh}[3 d]$$

$$\operatorname{Erf}\left[\frac{6 f x-b \operatorname{Log}[f]-2 c x \operatorname{Log}[f]}{2 \sqrt{3 f-c \operatorname{Log}[f]}}\right] \sqrt{3 f-c \operatorname{Log}[f]}+c\, e^{\frac{b^2 \operatorname{Log}[f]^2}{4 (3 f-c \operatorname{Log}[f])}} f^2 \operatorname{Cosh}[3 d] \operatorname{Erf}\left[\frac{6 f x-b \operatorname{Log}[f]-2 c x \operatorname{Log}[f]}{2 \sqrt{3 f-c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{3 f-c \operatorname{Log}[f]}-$$

$$3\,c^2\,e^{\frac{b^2\,\mathrm{Log}[f]^2}{4\,(3\,f-c\,\mathrm{Log}[f])}}\,f\,\mathrm{Cosh}[3\,d]\,\mathrm{Erf}\Big[\frac{6\,f\,x-b\,\mathrm{Log}[f]-2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{3\,f-c\,\mathrm{Log}[f]}}\Big]\,\mathrm{Log}[f]^2\,\sqrt{3\,f-c\,\mathrm{Log}[f]}\;-$$

$$c^3\,e^{\frac{b^2\,\mathrm{Log}[f]^2}{4\,(3\,f-c\,\mathrm{Log}[f])}}\,\mathrm{Cosh}[3\,d]\,\mathrm{Erf}\Big[\frac{6\,f\,x-b\,\mathrm{Log}[f]-2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{3\,f-c\,\mathrm{Log}[f]}}\Big]\,\mathrm{Log}[f]^3\,\sqrt{3\,f-c\,\mathrm{Log}[f]}\;+$$

$$27\,e^{-\frac{b^2\,\mathrm{Log}[f]^2}{4\,(f+c\,\mathrm{Log}[f])}}\,f^3\,\mathrm{Cosh}[d]\,\mathrm{Erfi}\Big[\frac{2\,f\,x+b\,\mathrm{Log}[f]+2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{f+c\,\mathrm{Log}[f]}}\Big]\,\sqrt{f+c\,\mathrm{Log}[f]}\;-27\,c\,e^{-\frac{b^2\,\mathrm{Log}[f]^2}{4\,(f+c\,\mathrm{Log}[f])}}\,f^2\,\mathrm{Cosh}[d]\,\mathrm{Erfi}\Big[\frac{2\,f\,x+b\,\mathrm{Log}[f]+2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{f+c\,\mathrm{Log}[f]}}\Big]$$

$$\mathrm{Log}[f]\,\sqrt{f+c\,\mathrm{Log}[f]}\;-3\,c^2\,e^{-\frac{b^2\,\mathrm{Log}[f]^2}{4\,(f+c\,\mathrm{Log}[f])}}\,f\,\mathrm{Cosh}[d]\,\mathrm{Erfi}\Big[\frac{2\,f\,x+b\,\mathrm{Log}[f]+2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{f+c\,\mathrm{Log}[f]}}\Big]\,\mathrm{Log}[f]^2\,\sqrt{f+c\,\mathrm{Log}[f]}\;+$$

$$3\,c^3\,e^{-\frac{b^2\,\mathrm{Log}[f]^2}{4\,(f+c\,\mathrm{Log}[f])}}\,\mathrm{Cosh}[d]\,\mathrm{Erfi}\Big[\frac{2\,f\,x+b\,\mathrm{Log}[f]+2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{f+c\,\mathrm{Log}[f]}}\Big]\,\mathrm{Log}[f]^3\,\sqrt{f+c\,\mathrm{Log}[f]}\;+$$

$$3\,e^{-\frac{b^2\,\mathrm{Log}[f]^2}{4\,(3\,f+c\,\mathrm{Log}[f])}}\,f^3\,\mathrm{Cosh}[3\,d]\,\mathrm{Erfi}\Big[\frac{6\,f\,x+b\,\mathrm{Log}[f]+2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{3\,f+c\,\mathrm{Log}[f]}}\Big]\,\sqrt{3\,f+c\,\mathrm{Log}[f]}\;-$$

$$c\,e^{-\frac{b^2\,\mathrm{Log}[f]^2}{4\,(3\,f+c\,\mathrm{Log}[f])}}\,f^2\,\mathrm{Cosh}[3\,d]\,\mathrm{Erfi}\Big[\frac{6\,f\,x+b\,\mathrm{Log}[f]+2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{3\,f+c\,\mathrm{Log}[f]}}\Big]\,\mathrm{Log}[f]\,\sqrt{3\,f+c\,\mathrm{Log}[f]}\;-$$

$$3\,c^2\,e^{-\frac{b^2\,\mathrm{Log}[f]^2}{4\,(3\,f+c\,\mathrm{Log}[f])}}\,f\,\mathrm{Cosh}[3\,d]\,\mathrm{Erfi}\Big[\frac{6\,f\,x+b\,\mathrm{Log}[f]+2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{3\,f+c\,\mathrm{Log}[f]}}\Big]\,\mathrm{Log}[f]^2\,\sqrt{3\,f+c\,\mathrm{Log}[f]}\;+$$

$$c^3\,e^{-\frac{b^2\,\mathrm{Log}[f]^2}{4\,(3\,f+c\,\mathrm{Log}[f])}}\,\mathrm{Cosh}[3\,d]\,\mathrm{Erfi}\Big[\frac{6\,f\,x+b\,\mathrm{Log}[f]+2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{3\,f+c\,\mathrm{Log}[f]}}\Big]\,\mathrm{Log}[f]^3\,\sqrt{3\,f+c\,\mathrm{Log}[f]}\;-$$

$$27\,e^{\frac{b^2\,\mathrm{Log}[f]^2}{4\,(f-c\,\mathrm{Log}[f])}}\,f^3\,\mathrm{Erf}\Big[\frac{2\,f\,x-b\,\mathrm{Log}[f]-2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{f-c\,\mathrm{Log}[f]}}\Big]\,\sqrt{f-c\,\mathrm{Log}[f]}\;\mathrm{Sinh}[d]\;-$$

$$27\,c\,e^{\frac{b^2\,\mathrm{Log}[f]^2}{4\,(f-c\,\mathrm{Log}[f])}}\,f^2\,\mathrm{Erf}\Big[\frac{2\,f\,x-b\,\mathrm{Log}[f]-2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{f-c\,\mathrm{Log}[f]}}\Big]\,\mathrm{Log}[f]\,\sqrt{f-c\,\mathrm{Log}[f]}\;\mathrm{Sinh}[d]\;+$$

$$3\,c^2\,e^{\frac{b^2\,\mathrm{Log}[f]^2}{4\,(f-c\,\mathrm{Log}[f])}}\,f\,\mathrm{Erf}\Big[\frac{2\,f\,x-b\,\mathrm{Log}[f]-2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{f-c\,\mathrm{Log}[f]}}\Big]\,\mathrm{Log}[f]^2\,\sqrt{f-c\,\mathrm{Log}[f]}\;\mathrm{Sinh}[d]\;+$$

$$3\,c^3\,e^{\frac{b^2\,\mathrm{Log}[f]^2}{4\,(f-c\,\mathrm{Log}[f])}}\,\mathrm{Erf}\Big[\frac{2\,f\,x-b\,\mathrm{Log}[f]-2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{f-c\,\mathrm{Log}[f]}}\Big]\,\mathrm{Log}[f]^3\,\sqrt{f-c\,\mathrm{Log}[f]}\;\mathrm{Sinh}[d]\;+$$

$$27\,e^{-\frac{b^2\,\mathrm{Log}[f]^2}{4\,(f+c\,\mathrm{Log}[f])}}\,f^3\,\mathrm{Erfi}\Big[\frac{2\,f\,x+b\,\mathrm{Log}[f]+2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{f+c\,\mathrm{Log}[f]}}\Big]\,\sqrt{f+c\,\mathrm{Log}[f]}\;\mathrm{Sinh}[d]\;-$$

$$27\,c\,e^{-\frac{b^2\,\mathrm{Log}[f]^2}{4\,(f+c\,\mathrm{Log}[f])}}\,f^2\,\mathrm{Erfi}\Big[\frac{2\,f\,x+b\,\mathrm{Log}[f]+2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{f+c\,\mathrm{Log}[f]}}\Big]\,\mathrm{Log}[f]\,\sqrt{f+c\,\mathrm{Log}[f]}\;\mathrm{Sinh}[d]\;-$$

$$3\,c^2\,e^{-\frac{b^2\,\mathrm{Log}[f]^2}{4\,(f+c\,\mathrm{Log}[f])}}\,f\,\mathrm{Erfi}\left[\frac{2\,f\,x+b\,\mathrm{Log}[f]+2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{f+c\,\mathrm{Log}[f]}}\right]\,\mathrm{Log}[f]^2\,\sqrt{f+c\,\mathrm{Log}[f]}\,\mathrm{Sinh}[d]+$$

$$3\,c^3\,e^{-\frac{b^2\,\mathrm{Log}[f]^2}{4\,(f+c\,\mathrm{Log}[f])}}\,\mathrm{Erfi}\left[\frac{2\,f\,x+b\,\mathrm{Log}[f]+2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{f+c\,\mathrm{Log}[f]}}\right]\,\mathrm{Log}[f]^3\,\sqrt{f+c\,\mathrm{Log}[f]}\,\mathrm{Sinh}[d]-$$

$$3\,e^{\frac{b^2\,\mathrm{Log}[f]^2}{4\,(3\,f-c\,\mathrm{Log}[f])}}\,f^3\,\mathrm{Erf}\left[\frac{6\,f\,x-b\,\mathrm{Log}[f]-2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{3\,f-c\,\mathrm{Log}[f]}}\right]\,\sqrt{3\,f-c\,\mathrm{Log}[f]}\,\mathrm{Sinh}[3\,d]-$$

$$c\,e^{\frac{b^2\,\mathrm{Log}[f]^2}{4\,(3\,f-c\,\mathrm{Log}[f])}}\,f^2\,\mathrm{Erf}\left[\frac{6\,f\,x-b\,\mathrm{Log}[f]-2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{3\,f-c\,\mathrm{Log}[f]}}\right]\,\mathrm{Log}[f]\,\sqrt{3\,f-c\,\mathrm{Log}[f]}\,\mathrm{Sinh}[3\,d]+$$

$$3\,c^2\,e^{\frac{b^2\,\mathrm{Log}[f]^2}{4\,(3\,f-c\,\mathrm{Log}[f])}}\,f\,\mathrm{Erf}\left[\frac{6\,f\,x-b\,\mathrm{Log}[f]-2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{3\,f-c\,\mathrm{Log}[f]}}\right]\,\mathrm{Log}[f]^2\,\sqrt{3\,f-c\,\mathrm{Log}[f]}\,\mathrm{Sinh}[3\,d]+$$

$$c^3\,e^{\frac{b^2\,\mathrm{Log}[f]^2}{4\,(3\,f-c\,\mathrm{Log}[f])}}\,\mathrm{Erf}\left[\frac{6\,f\,x-b\,\mathrm{Log}[f]-2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{3\,f-c\,\mathrm{Log}[f]}}\right]\,\mathrm{Log}[f]^3\,\sqrt{3\,f-c\,\mathrm{Log}[f]}\,\mathrm{Sinh}[3\,d]+$$

$$3\,e^{-\frac{b^2\,\mathrm{Log}[f]^2}{4\,(3\,f+c\,\mathrm{Log}[f])}}\,f^3\,\mathrm{Erfi}\left[\frac{6\,f\,x+b\,\mathrm{Log}[f]+2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{3\,f+c\,\mathrm{Log}[f]}}\right]\,\sqrt{3\,f+c\,\mathrm{Log}[f]}\,\mathrm{Sinh}[3\,d]-$$

$$c\,e^{-\frac{b^2\,\mathrm{Log}[f]^2}{4\,(3\,f+c\,\mathrm{Log}[f])}}\,f^2\,\mathrm{Erfi}\left[\frac{6\,f\,x+b\,\mathrm{Log}[f]+2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{3\,f+c\,\mathrm{Log}[f]}}\right]\,\mathrm{Log}[f]\,\sqrt{3\,f+c\,\mathrm{Log}[f]}\,\mathrm{Sinh}[3\,d]-$$

$$3\,c^2\,e^{-\frac{b^2\,\mathrm{Log}[f]^2}{4\,(3\,f+c\,\mathrm{Log}[f])}}\,f\,\mathrm{Erfi}\left[\frac{6\,f\,x+b\,\mathrm{Log}[f]+2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{3\,f+c\,\mathrm{Log}[f]}}\right]\,\mathrm{Log}[f]^2\,\sqrt{3\,f+c\,\mathrm{Log}[f]}\,\mathrm{Sinh}[3\,d]+$$

$$c^3\,e^{-\frac{b^2\,\mathrm{Log}[f]^2}{4\,(3\,f+c\,\mathrm{Log}[f])}}\,\mathrm{Erfi}\left[\frac{6\,f\,x+b\,\mathrm{Log}[f]+2\,c\,x\,\mathrm{Log}[f]}{2\,\sqrt{3\,f+c\,\mathrm{Log}[f]}}\right]\,\mathrm{Log}[f]^3\,\sqrt{3\,f+c\,\mathrm{Log}[f]}\,\mathrm{Sinh}[3\,d]\Bigg)$$

## Problem 327: Result more than twice size of optimal antiderivative.

$$\int f^{a+b\,x+c\,x^2}\,\mathrm{Cosh}\left[d+e\,x+f\,x^2\right]^2\,dx$$

Optimal (type 4, 239 leaves, 10 steps):

$$\frac{f^{a-\frac{b^2}{4c}}\,\sqrt{\pi}\,\mathrm{Erfi}\left[\frac{(b+2\,c\,x)\,\sqrt{\mathrm{Log}[f]}}{2\,\sqrt{c}}\right]}{4\,\sqrt{c}\,\sqrt{\mathrm{Log}[f]}}+\frac{e^{-2\,d+\frac{(2\,e-b\,\mathrm{Log}[f])^2}{8\,f-4\,c\,\mathrm{Log}[f]}}\,f^a\,\sqrt{\pi}\,\mathrm{Erf}\left[\frac{2\,e-b\,\mathrm{Log}[f]+2\,x\,(2\,f-c\,\mathrm{Log}[f])}{2\,\sqrt{2\,f-c\,\mathrm{Log}[f]}}\right]}{8\,\sqrt{2\,f-c\,\mathrm{Log}[f]}}+\frac{e^{2\,d-\frac{(2\,e+b\,\mathrm{Log}[f])^2}{8\,f+4\,c\,\mathrm{Log}[f]}}\,f^a\,\sqrt{\pi}\,\mathrm{Erfi}\left[\frac{2\,e+b\,\mathrm{Log}[f]+2\,x\,(2\,f+c\,\mathrm{Log}[f])}{2\,\sqrt{2\,f+c\,\mathrm{Log}[f]}}\right]}{8\,\sqrt{2\,f+c\,\mathrm{Log}[f]}}$$

Result (type 4, 912 leaves):

$$\frac{1}{8\,c\,\text{Log}[f]\,\left(2\,f-c\,\text{Log}[f]\right)\,\left(2\,f+c\,\text{Log}[f]\right)}$$

$$f^a\,\sqrt{\pi}\,\left(8\,\sqrt{c}\,f^{2-\frac{b^2}{4\,c}}\,\text{Erfi}\left[\frac{(b+2\,c\,x)\,\sqrt{\text{Log}[f]}}{2\,\sqrt{c}}\right]\,\sqrt{\text{Log}[f]}-2\,c^{5/2}\,f^{-\frac{b^2}{4\,c}}\,\text{Erfi}\left[\frac{(b+2\,c\,x)\,\sqrt{\text{Log}[f]}}{2\,\sqrt{c}}\right]\,\text{Log}[f]^{5/2}+\right.$$

$$2\,c\,e^{-\frac{-4\,e^2+4\,b\,e\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4\,(2\,f-c\,\text{Log}[f])}}\,f\,\text{Cosh}[2\,d]\,\text{Erf}\left[\frac{2\,e+4\,f\,x-b\,\text{Log}[f]-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{2\,f-c\,\text{Log}[f]}}\right]\,\text{Log}[f]\,\sqrt{2\,f-c\,\text{Log}[f]}+$$

$$c^2\,e^{-\frac{-4\,e^2+4\,b\,e\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4\,(2\,f-c\,\text{Log}[f])}}\,\text{Cosh}[2\,d]\,\text{Erf}\left[\frac{2\,e+4\,f\,x-b\,\text{Log}[f]-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{2\,f-c\,\text{Log}[f]}}\right]\,\text{Log}[f]^2\,\sqrt{2\,f-c\,\text{Log}[f]}+$$

$$2\,c\,e^{-\frac{4\,e^2+4\,b\,e\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4\,(2\,f+c\,\text{Log}[f])}}\,f\,\text{Cosh}[2\,d]\,\text{Erfi}\left[\frac{2\,e+4\,f\,x+b\,\text{Log}[f]+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{2\,f+c\,\text{Log}[f]}}\right]\,\text{Log}[f]\,\sqrt{2\,f+c\,\text{Log}[f]}-$$

$$c^2\,e^{-\frac{4\,e^2+4\,b\,e\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4\,(2\,f+c\,\text{Log}[f])}}\,\text{Cosh}[2\,d]\,\text{Erfi}\left[\frac{2\,e+4\,f\,x+b\,\text{Log}[f]+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{2\,f+c\,\text{Log}[f]}}\right]\,\text{Log}[f]^2\,\sqrt{2\,f+c\,\text{Log}[f]}-$$

$$2\,c\,e^{-\frac{-4\,e^2+4\,b\,e\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4\,(2\,f-c\,\text{Log}[f])}}\,f\,\text{Erf}\left[\frac{2\,e+4\,f\,x-b\,\text{Log}[f]-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{2\,f-c\,\text{Log}[f]}}\right]\,\text{Log}[f]\,\sqrt{2\,f-c\,\text{Log}[f]}\,\text{Sinh}[2\,d]-$$

$$c^2\,e^{-\frac{-4\,e^2+4\,b\,e\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4\,(2\,f-c\,\text{Log}[f])}}\,\text{Erf}\left[\frac{2\,e+4\,f\,x-b\,\text{Log}[f]-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{2\,f-c\,\text{Log}[f]}}\right]\,\text{Log}[f]^2\,\sqrt{2\,f-c\,\text{Log}[f]}\,\text{Sinh}[2\,d]+$$

$$2\,c\,e^{-\frac{4\,e^2+4\,b\,e\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4\,(2\,f+c\,\text{Log}[f])}}\,f\,\text{Erfi}\left[\frac{2\,e+4\,f\,x+b\,\text{Log}[f]+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{2\,f+c\,\text{Log}[f]}}\right]\,\text{Log}[f]\,\sqrt{2\,f+c\,\text{Log}[f]}\,\text{Sinh}[2\,d]-$$

$$\left.c^2\,e^{-\frac{4\,e^2+4\,b\,e\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4\,(2\,f+c\,\text{Log}[f])}}\,\text{Erfi}\left[\frac{2\,e+4\,f\,x+b\,\text{Log}[f]+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{2\,f+c\,\text{Log}[f]}}\right]\,\text{Log}[f]^2\,\sqrt{2\,f+c\,\text{Log}[f]}\,\text{Sinh}[2\,d]\right)$$

## Problem 328: Result more than twice size of optimal antiderivative.

$$\int f^{a+b\,x+c\,x^2}\,\text{Cosh}\left[d+e\,x+f\,x^2\right]^3\,dx$$

Optimal (type 4, 344 leaves, 14 steps) :

$$\frac{3\,e^{-d+\frac{(e-b\,\text{Log}[f])^2}{4\,(f-c\,\text{Log}[f])}}\,f^a\,\sqrt{\pi}\,\text{Erf}\left[\frac{e-b\,\text{Log}[f]+2\,x\,(f-c\,\text{Log}[f])}{2\,\sqrt{f-c\,\text{Log}[f]}}\right]}{16\,\sqrt{f-c\,\text{Log}[f]}}+\frac{e^{-3\,d+\frac{(3\,e-b\,\text{Log}[f])^2}{12\,f-4\,c\,\text{Log}[f]}}\,f^a\,\sqrt{\pi}\,\text{Erf}\left[\frac{3\,e-b\,\text{Log}[f]+2\,x\,(3\,f-c\,\text{Log}[f])}{2\,\sqrt{3\,f-c\,\text{Log}[f]}}\right]}{16\,\sqrt{3\,f-c\,\text{Log}[f]}}+$$

$$\frac{3\,e^{d-\frac{(e+b\,\text{Log}[f])^2}{4\,(f+c\,\text{Log}[f])}}\,f^a\,\sqrt{\pi}\,\text{Erfi}\left[\frac{e+b\,\text{Log}[f]+2\,x\,(f+c\,\text{Log}[f])}{2\,\sqrt{f+c\,\text{Log}[f]}}\right]}{16\,\sqrt{f+c\,\text{Log}[f]}}+\frac{e^{3\,d-\frac{(3\,e+b\,\text{Log}[f])^2}{4\,(3\,f+c\,\text{Log}[f])}}\,f^a\,\sqrt{\pi}\,\text{Erfi}\left[\frac{3\,e+b\,\text{Log}[f]+2\,x\,(3\,f+c\,\text{Log}[f])}{2\,\sqrt{3\,f+c\,\text{Log}[f]}}\right]}{16\,\sqrt{3\,f+c\,\text{Log}[f]}}$$

Result (type 4, 2991 leaves) :

$$\frac{1}{16\,(f-c\,\text{Log}[f])\,(3\,f-c\,\text{Log}[f])\,(f+c\,\text{Log}[f])\,(3\,f+c\,\text{Log}[f])}$$

$$f^a\,\sqrt{\pi}\,\Bigg(27\,e^{-\frac{-e^2+2\,b\,e\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4\,(f-c\,\text{Log}[f])}}\,f^3\,\text{Cosh}[d]\,\text{Erf}\left[\frac{e+2\,f\,x-b\,\text{Log}[f]-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f-c\,\text{Log}[f]}}\right]\,\sqrt{f-c\,\text{Log}[f]}\,+$$

$$27\,c\,e^{-\frac{-e^2+2\,b\,e\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4\,(f-c\,\text{Log}[f])}}\,f^2\,\text{Cosh}[d]\,\text{Erf}\left[\frac{e+2\,f\,x-b\,\text{Log}[f]-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f-c\,\text{Log}[f]}}\right]\,\text{Log}[f]\,\sqrt{f-c\,\text{Log}[f]}\,-$$

$$3\,c^2\,e^{-\frac{-e^2+2\,b\,e\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4\,(f-c\,\text{Log}[f])}}\,f\,\text{Cosh}[d]\,\text{Erf}\left[\frac{e+2\,f\,x-b\,\text{Log}[f]-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f-c\,\text{Log}[f]}}\right]\,\text{Log}[f]^2\,\sqrt{f-c\,\text{Log}[f]}\,-$$

$$3\,c^3\,e^{-\frac{-e^2+2\,b\,e\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4\,(f-c\,\text{Log}[f])}}\,\text{Cosh}[d]\,\text{Erf}\left[\frac{e+2\,f\,x-b\,\text{Log}[f]-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f-c\,\text{Log}[f]}}\right]\,\text{Log}[f]^3\,\sqrt{f-c\,\text{Log}[f]}\,+$$

$$3\,e^{-\frac{-9\,e^2+6\,b\,e\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4\,(3\,f-c\,\text{Log}[f])}}\,f^3\,\text{Cosh}[3\,d]\,\text{Erf}\left[\frac{3\,e+6\,f\,x-b\,\text{Log}[f]-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f-c\,\text{Log}[f]}}\right]\,\sqrt{3\,f-c\,\text{Log}[f]}\,+$$

$$c\,e^{-\frac{-9\,e^2+6\,b\,e\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4\,(3\,f-c\,\text{Log}[f])}}\,f^2\,\text{Cosh}[3\,d]\,\text{Erf}\left[\frac{3\,e+6\,f\,x-b\,\text{Log}[f]-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f-c\,\text{Log}[f]}}\right]\,\text{Log}[f]\,\sqrt{3\,f-c\,\text{Log}[f]}\,-$$

$$3\,c^2\,e^{-\frac{-9\,e^2+6\,b\,e\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4\,(3\,f-c\,\text{Log}[f])}}\,f\,\text{Cosh}[3\,d]\,\text{Erf}\left[\frac{3\,e+6\,f\,x-b\,\text{Log}[f]-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f-c\,\text{Log}[f]}}\right]\,\text{Log}[f]^2\,\sqrt{3\,f-c\,\text{Log}[f]}\,-$$

$$c^3\,e^{-\frac{-9\,e^2+6\,b\,e\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4\,(3\,f-c\,\text{Log}[f])}}\,\text{Cosh}[3\,d]\,\text{Erf}\left[\frac{3\,e+6\,f\,x-b\,\text{Log}[f]-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f-c\,\text{Log}[f]}}\right]\,\text{Log}[f]^3\,\sqrt{3\,f-c\,\text{Log}[f]}\,+$$

$$27\,e^{-\frac{e^2+2\,b\,e\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4\,(f+c\,\text{Log}[f])}}\,f^3\,\text{Cosh}[d]\,\text{Erfi}\left[\frac{e+2\,f\,x+b\,\text{Log}[f]+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f+c\,\text{Log}[f]}}\right]\,\sqrt{f+c\,\text{Log}[f]}\,-$$

$$27\,c\,e^{-\frac{e^2+2\,b\,e\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4\,(f+c\,\text{Log}[f])}}\,f^2\,\text{Cosh}[d]\,\text{Erfi}\left[\frac{e+2\,f\,x+b\,\text{Log}[f]+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f+c\,\text{Log}[f]}}\right]\,\text{Log}[f]\,\sqrt{f+c\,\text{Log}[f]}\,-$$

$$3\,c^2\,e^{-\frac{e^2+2\,b\,e\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4\,(f+c\,\text{Log}[f])}}\,f\,\text{Cosh}[d]\,\text{Erfi}\left[\frac{e+2\,f\,x+b\,\text{Log}[f]+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f+c\,\text{Log}[f]}}\right]\,\text{Log}[f]^2\,\sqrt{f+c\,\text{Log}[f]}\,+$$

$$3\,c^3\,e^{-\frac{e^2+2\,b\,e\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4\,(f+c\,\text{Log}[f])}}\,\text{Cosh}[d]\,\text{Erfi}\left[\frac{e+2\,f\,x+b\,\text{Log}[f]+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f+c\,\text{Log}[f]}}\right]\,\text{Log}[f]^3\,\sqrt{f+c\,\text{Log}[f]}\,+$$

$$3\,e^{-\frac{9\,e^2+6\,b\,e\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4\,(3\,f+c\,\text{Log}[f])}}\,f^3\,\text{Cosh}[3\,d]\,\text{Erfi}\left[\frac{3\,e+6\,f\,x+b\,\text{Log}[f]+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f+c\,\text{Log}[f]}}\right]\,\sqrt{3\,f+c\,\text{Log}[f]}\,-$$

$$c\,e^{-\frac{9\,e^2+6\,b\,e\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4\,(3\,f+c\,\text{Log}[f])}}\,f^2\,\text{Cosh}[3\,d]\,\text{Erfi}\left[\frac{3\,e+6\,f\,x+b\,\text{Log}[f]+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f+c\,\text{Log}[f]}}\right]\,\text{Log}[f]\,\sqrt{3\,f+c\,\text{Log}[f]}\,-$$

$$3\,c^2\,e^{-\frac{9\,e^2+6\,b\,e\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4\,(3\,f+c\,\text{Log}[f])}}\,f\,\text{Cosh}[3\,d]\,\text{Erfi}\left[\frac{3\,e+6\,f\,x+b\,\text{Log}[f]+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f+c\,\text{Log}[f]}}\right]\,\text{Log}[f]^2\,\sqrt{3\,f+c\,\text{Log}[f]}\,+$$

$$c^3\,e^{-\frac{9\,e^2+6\,b\,e\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4\,(3\,f+c\,\text{Log}[f])}}\,\text{Cosh}[3\,d]\,\text{Erfi}\left[\frac{3\,e+6\,f\,x+b\,\text{Log}[f]+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f+c\,\text{Log}[f]}}\right]\,\text{Log}[f]^3\,\sqrt{3\,f+c\,\text{Log}[f]}\,-$$

$$27\,e^{-\frac{-e^2+2\,b\,e\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4\,(f-c\,\text{Log}[f])}}\,f^3\,\text{Erf}\left[\frac{e+2\,f\,x-b\,\text{Log}[f]-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f-c\,\text{Log}[f]}}\right]\,\sqrt{f-c\,\text{Log}[f]}\,\text{Sinh}[d]\,-$$

$$27\,c\,e^{-\frac{-e^2+2\,b\,e\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4\,(f-c\,\text{Log}[f])}}\,f^2\,\text{Erf}\left[\frac{e+2\,f\,x-b\,\text{Log}[f]-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f-c\,\text{Log}[f]}}\right]\,\text{Log}[f]\,\sqrt{f-c\,\text{Log}[f]}\,\text{Sinh}[d]\,+$$

$$3\,c^2\,e^{-\frac{-e^2+2\,b\,e\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4\,(f-c\,\text{Log}[f])}}\,f\,\text{Erf}\left[\frac{e+2\,f\,x-b\,\text{Log}[f]-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f-c\,\text{Log}[f]}}\right]\,\text{Log}[f]^2\,\sqrt{f-c\,\text{Log}[f]}\,\text{Sinh}[d]\,+$$

$$3\,c^3\,e^{-\frac{-e^2+2\,b\,e\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4\,(f-c\,\text{Log}[f])}}\,\text{Erf}\left[\frac{e+2\,f\,x-b\,\text{Log}[f]-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f-c\,\text{Log}[f]}}\right]\,\text{Log}[f]^3\,\sqrt{f-c\,\text{Log}[f]}\,\text{Sinh}[d]\,+$$

$$27\,e^{-\frac{e^2+2\,b\,e\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4\,(f+c\,\text{Log}[f])}}\,f^3\,\text{Erfi}\left[\frac{e+2\,f\,x+b\,\text{Log}[f]+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f+c\,\text{Log}[f]}}\right]\,\sqrt{f+c\,\text{Log}[f]}\,\text{Sinh}[d]\,-$$

$$27\,c\,e^{-\frac{e^2+2\,b\,e\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4\,(f+c\,\text{Log}[f])}}\,f^2\,\text{Erfi}\left[\frac{e+2\,f\,x+b\,\text{Log}[f]+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f+c\,\text{Log}[f]}}\right]\,\text{Log}[f]\,\sqrt{f+c\,\text{Log}[f]}\,\text{Sinh}[d]\,-$$

$$3\,c^2\,e^{-\frac{e^2+2\,b\,e\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4\,(f+c\,\text{Log}[f])}}\,f\,\text{Erfi}\left[\frac{e+2\,f\,x+b\,\text{Log}[f]+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f+c\,\text{Log}[f]}}\right]\,\text{Log}[f]^2\,\sqrt{f+c\,\text{Log}[f]}\,\text{Sinh}[d]\,+$$

$$3\,c^3\,e^{-\frac{e^2+2\,b\,e\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4\,(f+c\,\text{Log}[f])}}\,\text{Erfi}\left[\frac{e+2\,f\,x+b\,\text{Log}[f]+2\,c\,x\,\text{Log}[f]}{2\,\sqrt{f+c\,\text{Log}[f]}}\right]\,\text{Log}[f]^3\,\sqrt{f+c\,\text{Log}[f]}\,\text{Sinh}[d]\,-$$

$$3\,e^{-\frac{-9\,e^2+6\,b\,e\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4\,(3\,f-c\,\text{Log}[f])}}\,f^3\,\text{Erf}\left[\frac{3\,e+6\,f\,x-b\,\text{Log}[f]-2\,c\,x\,\text{Log}[f]}{2\,\sqrt{3\,f-c\,\text{Log}[f]}}\right]\,\sqrt{3\,f-c\,\text{Log}[f]}\,\text{Sinh}[3\,d]\,-$$

$$c\,e^{-\frac{-9e^2+6be\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4(3f-c\,\text{Log}[f])}} f^2\,\text{Erf}\left[\frac{3e+6fx-b\,\text{Log}[f]-2cx\,\text{Log}[f]}{2\sqrt{3f-c\,\text{Log}[f]}}\right]\text{Log}[f]\sqrt{3f-c\,\text{Log}[f]}\,\text{Sinh}[3d]+$$

$$3c^2\,e^{-\frac{-9e^2+6be\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4(3f-c\,\text{Log}[f])}} f\,\text{Erf}\left[\frac{3e+6fx-b\,\text{Log}[f]-2cx\,\text{Log}[f]}{2\sqrt{3f-c\,\text{Log}[f]}}\right]\text{Log}[f]^2\sqrt{3f-c\,\text{Log}[f]}\,\text{Sinh}[3d]+$$

$$c^3\,e^{-\frac{-9e^2+6be\,\text{Log}[f]-b^2\,\text{Log}[f]^2}{4(3f-c\,\text{Log}[f])}}\,\text{Erf}\left[\frac{3e+6fx-b\,\text{Log}[f]-2cx\,\text{Log}[f]}{2\sqrt{3f-c\,\text{Log}[f]}}\right]\text{Log}[f]^3\sqrt{3f-c\,\text{Log}[f]}\,\text{Sinh}[3d]+$$

$$3\,e^{-\frac{9e^2+6be\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4(3f+c\,\text{Log}[f])}} f^3\,\text{Erfi}\left[\frac{3e+6fx+b\,\text{Log}[f]+2cx\,\text{Log}[f]}{2\sqrt{3f+c\,\text{Log}[f]}}\right]\sqrt{3f+c\,\text{Log}[f]}\,\text{Sinh}[3d]-$$

$$c\,e^{-\frac{9e^2+6be\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4(3f+c\,\text{Log}[f])}} f^2\,\text{Erfi}\left[\frac{3e+6fx+b\,\text{Log}[f]+2cx\,\text{Log}[f]}{2\sqrt{3f+c\,\text{Log}[f]}}\right]\text{Log}[f]\sqrt{3f+c\,\text{Log}[f]}\,\text{Sinh}[3d]-$$

$$3c^2\,e^{-\frac{9e^2+6be\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4(3f+c\,\text{Log}[f])}} f\,\text{Erfi}\left[\frac{3e+6fx+b\,\text{Log}[f]+2cx\,\text{Log}[f]}{2\sqrt{3f+c\,\text{Log}[f]}}\right]\text{Log}[f]^2\sqrt{3f+c\,\text{Log}[f]}\,\text{Sinh}[3d]+$$

$$c^3\,e^{-\frac{9e^2+6be\,\text{Log}[f]+b^2\,\text{Log}[f]^2}{4(3f+c\,\text{Log}[f])}}\,\text{Erfi}\left[\frac{3e+6fx+b\,\text{Log}[f]+2cx\,\text{Log}[f]}{2\sqrt{3f+c\,\text{Log}[f]}}\right]\text{Log}[f]^3\sqrt{3f+c\,\text{Log}[f]}\,\text{Sinh}[3d]\Bigg)$$

## Problem 329: Result more than twice size of optimal antiderivative.

$$\int\left(\frac{x}{\text{Cosh}[x]^{3/2}}+x\sqrt{\text{Cosh}[x]}\right)dx$$

Optimal (type 3, 20 leaves, 2 steps):

$$-4\sqrt{\text{Cosh}[x]}+\frac{2x\,\text{Sinh}[x]}{\sqrt{\text{Cosh}[x]}}$$

Result (type 3, 46 leaves):

$$\frac{2\,\text{Sinh}[x]\left(x-\frac{2\,\text{Cosh}[x]\,\text{Sinh}[x]\sqrt{\text{Tanh}\left[\frac{x}{2}\right]^2}}{(-1+\text{Cosh}[x])^{3/2}\sqrt{1+\text{Cosh}[x]}}\right)}{\sqrt{\text{Cosh}[x]}}$$

## Problem 332: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\left(\frac{x^2}{\text{Cosh}[x]^{3/2}}+x^2\sqrt{\text{Cosh}[x]}\right)dx$$

Optimal (type 4, 36 leaves, 3 steps):

$$-8\,x\,\sqrt{\text{Cosh}[x]}-16\,i\,\text{EllipticE}\left[\frac{i\,x}{2},2\right]+\frac{2\,x^2\,\text{Sinh}[x]}{\sqrt{\text{Cosh}[x]}}$$

Result (type 5, 76 leaves):

$$\frac{1}{1+e^{2\,x}}4\,\sqrt{\text{Cosh}[x]}\,\left(\text{Cosh}[x]+\text{Sinh}[x]\right)$$
$$\left(-4\,\left(-2+x\right)\,\text{Cosh}[x]+x^2\,\text{Sinh}[x]+8\,\text{Hypergeometric2F1}\left[-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2\,x}\right]\,\left(-\text{Cosh}[x]+\text{Sinh}[x]\right)\,\sqrt{1+\text{Cosh}[2\,x]+\text{Sinh}[2\,x]}\right)$$

## Problem 335: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{Cosh}[a+b\,x]}{c+d\,x^2}\,dx$$

Optimal (type 4, 213 leaves, 8 steps):

$$\frac{\text{Cosh}\left[a+\frac{b\sqrt{-c}}{\sqrt{d}}\right]\,\text{CoshIntegral}\left[\frac{b\sqrt{-c}}{\sqrt{d}}-b\,x\right]}{2\,\sqrt{-c}\,\sqrt{d}}-\frac{\text{Cosh}\left[a-\frac{b\sqrt{-c}}{\sqrt{d}}\right]\,\text{CoshIntegral}\left[\frac{b\sqrt{-c}}{\sqrt{d}}+b\,x\right]}{2\,\sqrt{-c}\,\sqrt{d}}-$$
$$\frac{\text{Sinh}\left[a+\frac{b\sqrt{-c}}{\sqrt{d}}\right]\,\text{SinhIntegral}\left[\frac{b\sqrt{-c}}{\sqrt{d}}-b\,x\right]}{2\,\sqrt{-c}\,\sqrt{d}}-\frac{\text{Sinh}\left[a-\frac{b\sqrt{-c}}{\sqrt{d}}\right]\,\text{SinhIntegral}\left[\frac{b\sqrt{-c}}{\sqrt{d}}+b\,x\right]}{2\,\sqrt{-c}\,\sqrt{d}}$$

Result (type 4, 180 leaves):

$$\frac{1}{2\,\sqrt{c}\,\sqrt{d}}i\,\left(\text{Cosh}\left[a-\frac{i\,b\,\sqrt{c}}{\sqrt{d}}\right]\,\text{CosIntegral}\left[-\frac{b\,\sqrt{c}}{\sqrt{d}}+i\,b\,x\right]-\text{Cosh}\left[a+\frac{i\,b\,\sqrt{c}}{\sqrt{d}}\right]\,\text{CosIntegral}\left[\frac{b\,\sqrt{c}}{\sqrt{d}}+i\,b\,x\right]+\right.$$
$$\left.i\,\left(\text{Sinh}\left[a-\frac{i\,b\,\sqrt{c}}{\sqrt{d}}\right]\,\text{SinIntegral}\left[\frac{b\,\sqrt{c}}{\sqrt{d}}-i\,b\,x\right]+\text{Sinh}\left[a+\frac{i\,b\,\sqrt{c}}{\sqrt{d}}\right]\,\text{SinIntegral}\left[\frac{b\,\sqrt{c}}{\sqrt{d}}+i\,b\,x\right]\right)\right)$$

## Problem 336: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Cosh}[a+b\,x]}{c+d\,x+e\,x^2}\,dx$$

Optimal (type 4, 271 leaves, 8 steps):

$$\frac{\text{Cosh}\left[a-\frac{b\left(d-\sqrt{d^2-4\,c\,e}\right)}{2\,e}\right]\,\text{CoshIntegral}\left[\frac{b\left(d-\sqrt{d^2-4\,c\,e}\right)}{2\,e}+b\,x\right]}{\sqrt{d^2-4\,c\,e}}-\frac{\text{Cosh}\left[a-\frac{b\left(d+\sqrt{d^2-4\,c\,e}\right)}{2\,e}\right]\,\text{CoshIntegral}\left[\frac{b\left(d+\sqrt{d^2-4\,c\,e}\right)}{2\,e}+b\,x\right]}{\sqrt{d^2-4\,c\,e}}+$$
$$\frac{\text{Sinh}\left[a-\frac{b\left(d-\sqrt{d^2-4\,c\,e}\right)}{2\,e}\right]\,\text{SinhIntegral}\left[\frac{b\left(d-\sqrt{d^2-4\,c\,e}\right)}{2\,e}+b\,x\right]}{\sqrt{d^2-4\,c\,e}}-\frac{\text{Sinh}\left[a-\frac{b\left(d+\sqrt{d^2-4\,c\,e}\right)}{2\,e}\right]\,\text{SinhIntegral}\left[\frac{b\left(d+\sqrt{d^2-4\,c\,e}\right)}{2\,e}+b\,x\right]}{\sqrt{d^2-4\,c\,e}}$$

Result (type 4, 248 leaves):

$$\frac{1}{\sqrt{d^2-4\,c\,e}}\left(\text{Cosh}\left[a+\frac{b\left(-d+\sqrt{d^2-4\,c\,e}\right)}{2\,e}\right]\,\text{CosIntegral}\left[\frac{i\,b\left(d-\sqrt{d^2-4\,c\,e}+2\,e\,x\right)}{2\,e}\right]-\right.$$
$$\text{Cosh}\left[a-\frac{b\left(d+\sqrt{d^2-4\,c\,e}\right)}{2\,e}\right]\,\text{CosIntegral}\left[\frac{i\,b\left(d+\sqrt{d^2-4\,c\,e}+2\,e\,x\right)}{2\,e}\right]-$$
$$\text{Sinh}\left[a-\frac{b\left(d+\sqrt{d^2-4\,c\,e}\right)}{2\,e}\right]\,\text{SinhIntegral}\left[\frac{b\left(d+\sqrt{d^2-4\,c\,e}+2\,e\,x\right)}{2\,e}\right]+$$
$$\left.i\,\text{Sinh}\left[a+\frac{b\left(-d+\sqrt{d^2-4\,c\,e}\right)}{2\,e}\right]\,\text{SinIntegral}\left[\frac{i\,b\left(-d+\sqrt{d^2-4\,c\,e}\right)}{2\,e}-i\,b\,x\right]\right)$$

# Test results for the 85 problems in "6.2.7 hyper^m (a+b cosh^n)^p.m"

## Problem 6: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Sinh}[x]^7}{a+b\,\text{Cosh}[x]^2}\,dx$$

Optimal (type 3, 78 leaves, 4 steps):

$$-\frac{(a+b)^3\,\text{ArcTan}\left[\frac{\sqrt{b}\,\text{Cosh}[x]}{\sqrt{a}}\right]}{\sqrt{a}\,b^{7/2}}+\frac{\left(a^2+3\,a\,b+3\,b^2\right)\,\text{Cosh}[x]}{b^3}-\frac{(a+3\,b)\,\text{Cosh}[x]^3}{3\,b^2}+\frac{\text{Cosh}[x]^5}{5\,b}$$

Result (type 3, 148 leaves):

$$-\frac{(a+b)^3 \text{ArcTan}\left[\frac{\sqrt{b}-i\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]}{\sqrt{a}\,b^{7/2}}-\frac{(a+b)^3 \text{ArcTan}\left[\frac{\sqrt{b}+i\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]}{\sqrt{a}\,b^{7/2}}+\frac{\left(8a^2+22ab+19b^2\right)\text{Cosh}[x]}{8b^3}-\frac{(4a+9b)\,\text{Cosh}[3x]}{48b^2}+\frac{\text{Cosh}[5x]}{80b}$$

## Problem 7: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sinh}[x]^5}{a+b\,\text{Cosh}[x]^2}\,dx$$

Optimal (type 3, 54 leaves, 4 steps):

$$\frac{(a+b)^2 \text{ArcTan}\left[\frac{\sqrt{b}\,\text{Cosh}[x]}{\sqrt{a}}\right]}{\sqrt{a}\,b^{5/2}}-\frac{(a+2b)\,\text{Cosh}[x]}{b^2}+\frac{\text{Cosh}[x]^3}{3b}$$

Result (type 3, 120 leaves):

$$\frac{1}{12b^{5/2}}\left(\frac{12(a+b)^2 \text{ArcTan}\left[\frac{\sqrt{b}-i\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]}{\sqrt{a}}+\frac{12(a+b)^2 \text{ArcTan}\left[\frac{\sqrt{b}+i\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]}{\sqrt{a}}-3\sqrt{b}\,(4a+7b)\,\text{Cosh}[x]+b^{3/2}\,\text{Cosh}[3x]\right)$$

## Problem 8: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sinh}[x]^3}{a+b\,\text{Cosh}[x]^2}\,dx$$

Optimal (type 3, 36 leaves, 3 steps):

$$-\frac{(a+b)\,\text{ArcTan}\left[\frac{\sqrt{b}\,\text{Cosh}[x]}{\sqrt{a}}\right]}{\sqrt{a}\,b^{3/2}}+\frac{\text{Cosh}[x]}{b}$$

Result (type 3, 83 leaves):

$$-\frac{(a+b)\left(\text{ArcTan}\left[\frac{\sqrt{b}-i\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]+\text{ArcTan}\left[\frac{\sqrt{b}+i\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]\right)}{\sqrt{a}\,b^{3/2}}+\frac{\text{Cosh}[x]}{b}$$

## Problem 10: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[x]}{a+b\,\text{Cosh}[x]^2}\,dx$$

Optimal (type 3, 42 leaves, 4 steps):

$$-\frac{\sqrt{b}\,\text{ArcTan}\left[\frac{\sqrt{b}\,\text{Cosh}[x]}{\sqrt{a}}\right]}{\sqrt{a}\,(a+b)}-\frac{\text{ArcTanh}[\text{Cosh}[x]]}{a+b}$$

Result (type 3, 106 leaves):

$$-\frac{\frac{\sqrt{b}\,\text{ArcTan}\left[\frac{\sqrt{b}-i\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]}{\sqrt{a}}+\frac{\sqrt{b}\,\text{ArcTan}\left[\frac{\sqrt{b}+i\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]}{\sqrt{a}}+\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]}{a+b}$$

## Problem 11: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[x]^3}{a+b\,\text{Cosh}[x]^2}\,dx$$

Optimal (type 3, 61 leaves, 5 steps):

$$\frac{b^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{b}\,\text{Cosh}[x]}{\sqrt{a}}\right]}{\sqrt{a}\,(a+b)^2}+\frac{(a+3\,b)\,\text{ArcTanh}[\text{Cosh}[x]]}{2\,(a+b)^2}-\frac{\text{Coth}[x]\,\text{Csch}[x]}{2\,(a+b)}$$

Result (type 3, 154 leaves):

$$\frac{1}{8\sqrt{a}\,(a+b)^2}\left(8\,b^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{b}-i\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]+8\,b^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{b}+i\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]-\right.$$
$$\left.\sqrt{a}\,(a+b)\,\text{Csch}\left[\frac{x}{2}\right]^2+4\sqrt{a}\,(a+3\,b)\left(\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]\right)-\sqrt{a}\,(a+b)\,\text{Sech}\left[\frac{x}{2}\right]^2\right)$$

## Problem 12: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[x]^5}{a+b\,\text{Cosh}[x]^2}\,dx$$

Optimal (type 3, 94 leaves, 6 steps):

$$-\frac{b^{5/2}\,\text{ArcTan}\left[\frac{\sqrt{b}\,\text{Cosh}[x]}{\sqrt{a}}\right]}{\sqrt{a}\,(a+b)^3}-\frac{\left(3a^2+10ab+15b^2\right)\text{ArcTanh}[\text{Cosh}[x]]}{8\,(a+b)^3}+\frac{(3a+7b)\,\text{Coth}[x]\,\text{Csch}[x]}{8\,(a+b)^2}-\frac{\text{Coth}[x]\,\text{Csch}[x]^3}{4\,(a+b)}$$

Result (type 3, 229 leaves):

$$\frac{1}{64\sqrt{a}\,(a+b)^3}\left(2\sqrt{a}\left(3a^2+10ab+7b^2\right)\text{Csch}\left[\frac{x}{2}\right]^2-\sqrt{a}\,(a+b)^2\,\text{Csch}\left[\frac{x}{2}\right]^4-8\left(8b^{5/2}\,\text{ArcTan}\left[\frac{\sqrt{b}-i\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]+8b^{5/2}\,\text{ArcTan}\left[\frac{\sqrt{b}+i\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]+\sqrt{a}\left(3a^2+10ab+15b^2\right)\left(\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]\right)\right)+2\sqrt{a}\left(3a^2+10ab+7b^2\right)\text{Sech}\left[\frac{x}{2}\right]^2+\sqrt{a}\,(a+b)^2\,\text{Sech}\left[\frac{x}{2}\right]^4\right)$$

## Problem 56: Result is not expressed in closed-form.

$$\int\frac{1}{a+b\,\text{Cosh}[x]^3}\,dx$$

Optimal (type 3, 288 leaves, 8 steps):

$$\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/3}-b^{1/3}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/3}+b^{1/3}}}\right]}{3a^{2/3}\sqrt{a^{1/3}-b^{1/3}}\sqrt{a^{1/3}+b^{1/3}}}+\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/3}+(-1)^{1/3}b^{1/3}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/3}-(-1)^{1/3}b^{1/3}}}\right]}{3a^{2/3}\sqrt{a^{1/3}-(-1)^{1/3}b^{1/3}}\sqrt{a^{1/3}+(-1)^{1/3}b^{1/3}}}+\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/3}-(-1)^{2/3}b^{1/3}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/3}+(-1)^{2/3}b^{1/3}}}\right]}{3a^{2/3}\sqrt{a^{1/3}-(-1)^{2/3}b^{1/3}}\sqrt{a^{1/3}+(-1)^{2/3}b^{1/3}}}$$

Result (type 7, 105 leaves):

$$\frac{2}{3}\,\text{RootSum}\left[b+3b\,\#1^2+8a\,\#1^3+3b\,\#1^4+b\,\#1^6\,\&,\ \frac{x\,\#1+2\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1}{b+4a\,\#1+2b\,\#1^2+b\,\#1^4}\,\&\right]$$

## Problem 57: Result is not expressed in closed-form.

$$\int\frac{1}{a-b\,\text{Cosh}[x]^3}\,dx$$

Optimal (type 3, 288 leaves, 8 steps):

$$\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/3}+b^{1/3}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/3}-b^{1/3}}}\right]}{3a^{2/3}\sqrt{a^{1/3}-b^{1/3}}\sqrt{a^{1/3}+b^{1/3}}}+\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/3}-(-1)^{1/3}b^{1/3}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/3}+(-1)^{1/3}b^{1/3}}}\right]}{3a^{2/3}\sqrt{a^{1/3}-(-1)^{1/3}b^{1/3}}\sqrt{a^{1/3}+(-1)^{1/3}b^{1/3}}}+\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/3}+(-1)^{2/3}b^{1/3}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/3}-(-1)^{2/3}b^{1/3}}}\right]}{3a^{2/3}\sqrt{a^{1/3}-(-1)^{2/3}b^{1/3}}\sqrt{a^{1/3}+(-1)^{2/3}b^{1/3}}}$$

Result (type 7, 105 leaves):

$$-\frac{2}{3}\,\text{RootSum}\Big[b+3\,b\,\#1^2-8\,a\,\#1^3+3\,b\,\#1^4+b\,\#1^6\,\&,\ \frac{x\,\#1+2\,\text{Log}\big[-\text{Cosh}\big[\frac{x}{2}\big]-\text{Sinh}\big[\frac{x}{2}\big]+\text{Cosh}\big[\frac{x}{2}\big]\,\#1-\text{Sinh}\big[\frac{x}{2}\big]\,\#1\big]\,\#1}{b-4\,a\,\#1+2\,b\,\#1^2+b\,\#1^4}\,\&\Big]$$

## Problem 60: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{a+b\,\text{Cosh}[x]^4}\,dx$$

Optimal (type 3, 361 leaves, 10 steps):

$$\frac{\sqrt{\sqrt{a}-\sqrt{a+b}}\ \text{ArcTanh}\Big[\frac{\sqrt{\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\,a^{1/4}\,\text{Tanh}[x]}{\sqrt{\sqrt{a}-\sqrt{a+b}}}\Big]}{2\,\sqrt{2}\,a^{3/4}\,\sqrt{a+b}}-\frac{\sqrt{\sqrt{a}-\sqrt{a+b}}\ \text{ArcTanh}\Big[\frac{\sqrt{\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\,a^{1/4}\,\text{Tanh}[x]}{\sqrt{\sqrt{a}-\sqrt{a+b}}}\Big]}{2\,\sqrt{2}\,a^{3/4}\,\sqrt{a+b}}-$$
$$\frac{\sqrt{\sqrt{a}+\sqrt{a+b}}\ \text{Log}\Big[\sqrt{a+b}-\sqrt{2}\,a^{1/4}\,\sqrt{\sqrt{a}+\sqrt{a+b}}\ \text{Tanh}[x]+\sqrt{a}\,\text{Tanh}[x]^2\Big]}{4\,\sqrt{2}\,a^{3/4}\,\sqrt{a+b}}+$$
$$\frac{\sqrt{\sqrt{a}+\sqrt{a+b}}\ \text{Log}\Big[\sqrt{a+b}+\sqrt{2}\,a^{1/4}\,\sqrt{\sqrt{a}+\sqrt{a+b}}\ \text{Tanh}[x]+\sqrt{a}\,\text{Tanh}[x]^2\Big]}{4\,\sqrt{2}\,a^{3/4}\,\sqrt{a+b}}$$

Result (type 3, 121 leaves):

$$-\frac{\text{ArcTan}\Big[\frac{\sqrt{a}\,\text{Tanh}[x]}{\sqrt{-a+i\,\sqrt{a}\,\sqrt{b}}}\Big]}{2\,\sqrt{a}\,\sqrt{-a+i\,\sqrt{a}\,\sqrt{b}}}+\frac{\text{ArcTanh}\Big[\frac{\sqrt{a}\,\text{Tanh}[x]}{\sqrt{a+i\,\sqrt{a}\,\sqrt{b}}}\Big]}{2\,\sqrt{a}\,\sqrt{a+i\,\sqrt{a}\,\sqrt{b}}}$$

## Problem 62: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{1+\text{Cosh}[x]^4}\,dx$$

Optimal (type 3, 176 leaves, 10 steps):

$$-\frac{\text{ArcTan}\Big[\frac{\sqrt{1+\sqrt{2}}-2\,\text{Coth}[x]}{\sqrt{-1+\sqrt{2}}}\Big]}{4\,\sqrt{1+\sqrt{2}}}+\frac{\text{ArcTan}\Big[\frac{\sqrt{1+\sqrt{2}}+2\,\text{Coth}[x]}{\sqrt{-1+\sqrt{2}}}\Big]}{4\,\sqrt{1+\sqrt{2}}}-$$
$$\frac{1}{8}\,\sqrt{1+\sqrt{2}}\ \text{Log}\Big[\sqrt{2}-2\,\sqrt{1+\sqrt{2}}\ \text{Coth}[x]+2\,\text{Coth}[x]^2\Big]+\frac{1}{8}\,\sqrt{1+\sqrt{2}}\ \text{Log}\Big[1+\sqrt{2\,\left(1+\sqrt{2}\right)}\ \text{Coth}[x]+\sqrt{2}\,\text{Coth}[x]^2\Big]$$

Result (type 3, 45 leaves):

$$\frac{\text{ArcTanh}\left[\frac{\text{Tanh}[x]}{\sqrt{1-i}}\right]}{2\sqrt{1-i}}+\frac{\text{ArcTanh}\left[\frac{\text{Tanh}[x]}{\sqrt{1+i}}\right]}{2\sqrt{1+i}}$$

## Problem 64: Result is not expressed in closed-form.

$$\int\frac{1}{a+b\,\text{Cosh}[x]^5}\,dx$$

Optimal (type 3, 494 leaves, 12 steps):

$$\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}-b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}+b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-b^{1/5}}\,\sqrt{a^{1/5}+b^{1/5}}}+\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}+(-1)^{1/5}b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}-(-1)^{1/5}b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-(-1)^{1/5}b^{1/5}}\,\sqrt{a^{1/5}+(-1)^{1/5}b^{1/5}}}+\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}-(-1)^{2/5}b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}+(-1)^{2/5}b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-(-1)^{2/5}b^{1/5}}\,\sqrt{a^{1/5}+(-1)^{2/5}b^{1/5}}}+$$

$$\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}+(-1)^{3/5}b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}-(-1)^{3/5}b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-(-1)^{3/5}b^{1/5}}\,\sqrt{a^{1/5}+(-1)^{3/5}b^{1/5}}}+\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}-(-1)^{4/5}b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}+(-1)^{4/5}b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-(-1)^{4/5}b^{1/5}}\,\sqrt{a^{1/5}+(-1)^{4/5}b^{1/5}}}$$

Result (type 7, 139 leaves):

$$\frac{8}{5}\,\text{RootSum}\left[b+5\,b\,\#1^2+10\,b\,\#1^4+32\,a\,\#1^5+10\,b\,\#1^6+5\,b\,\#1^8+b\,\#1^{10}\,\&,\ \frac{x\,\#1^3+2\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^3}{b+4\,b\,\#1^2+16\,a\,\#1^3+6\,b\,\#1^4+4\,b\,\#1^6+b\,\#1^8}\,\&\right]$$

## Problem 65: Result is not expressed in closed-form.

$$\int\frac{1}{a+b\,\text{Cosh}[x]^6}\,dx$$

Optimal (type 3, 171 leaves, 7 steps):

$$\frac{\text{ArcTanh}\left[\frac{a^{1/6}\,\text{Tanh}[x]}{\sqrt{a^{1/3}+b^{1/3}}}\right]}{3\,a^{5/6}\sqrt{a^{1/3}+b^{1/3}}}+\frac{\text{ArcTanh}\left[\frac{a^{1/6}\,\text{Tanh}[x]}{\sqrt{a^{1/3}-(-1)^{1/3}b^{1/3}}}\right]}{3\,a^{5/6}\sqrt{a^{1/3}-(-1)^{1/3}b^{1/3}}}+\frac{\text{ArcTanh}\left[\frac{a^{1/6}\,\text{Tanh}[x]}{\sqrt{a^{1/3}+(-1)^{2/3}b^{1/3}}}\right]}{3\,a^{5/6}\sqrt{a^{1/3}+(-1)^{2/3}b^{1/3}}}$$

Result (type 7, 132 leaves):

$$\frac{16}{3}\,\text{RootSum}\left[b+6\,b\,\#1+15\,b\,\#1^2+64\,a\,\#1^3+20\,b\,\#1^3+15\,b\,\#1^4+6\,b\,\#1^5+b\,\#1^6\,\&,\ \frac{x\,\#1^2+\text{Log}\left[-\text{Cosh}[x]-\text{Sinh}[x]+\text{Cosh}[x]\,\#1-\text{Sinh}[x]\,\#1\right]\#1^2}{b+5\,b\,\#1+32\,a\,\#1^2+10\,b\,\#1^2+10\,b\,\#1^3+5\,b\,\#1^4+b\,\#1^5}\,\&\right]$$

## Problem 66: Result is not expressed in closed-form.

$$\int \frac{1}{a+b\,\text{Cosh}[x]^8}\,dx$$

Optimal (type 3, 245 leaves, 9 steps):

$$-\frac{\text{ArcTanh}\left[\frac{(-a)^{1/8}\,\text{Tanh}[x]}{\sqrt{(-a)^{1/4}-b^{1/4}}}\right]}{4\,(-a)^{7/8}\sqrt{(-a)^{1/4}-b^{1/4}}}-\frac{\text{ArcTanh}\left[\frac{(-a)^{1/8}\,\text{Tanh}[x]}{\sqrt{(-a)^{1/4}-i\,b^{1/4}}}\right]}{4\,(-a)^{7/8}\sqrt{(-a)^{1/4}-i\,b^{1/4}}}-\frac{\text{ArcTanh}\left[\frac{(-a)^{1/8}\,\text{Tanh}[x]}{\sqrt{(-a)^{1/4}+i\,b^{1/4}}}\right]}{4\,(-a)^{7/8}\sqrt{(-a)^{1/4}+i\,b^{1/4}}}-\frac{\text{ArcTanh}\left[\frac{(-a)^{1/8}\,\text{Tanh}[x]}{\sqrt{(-a)^{1/4}+b^{1/4}}}\right]}{4\,(-a)^{7/8}\sqrt{(-a)^{1/4}+b^{1/4}}}$$

Result (type 7, 158 leaves):

$$16\,\text{RootSum}\Big[b+8\,b\,\#1+28\,b\,\#1^2+56\,b\,\#1^3+256\,a\,\#1^4+70\,b\,\#1^4+56\,b\,\#1^5+28\,b\,\#1^6+8\,b\,\#1^7+b\,\#1^8\,\&,$$
$$\frac{x\,\#1^3+\text{Log}[-\text{Cosh}[x]-\text{Sinh}[x]+\text{Cosh}[x]\,\#1-\text{Sinh}[x]\,\#1]\,\#1^3}{b+7\,b\,\#1+21\,b\,\#1^2+128\,a\,\#1^3+35\,b\,\#1^3+35\,b\,\#1^4+21\,b\,\#1^5+7\,b\,\#1^6+b\,\#1^7}\,\&\Big]$$

## Problem 67: Result is not expressed in closed-form.

$$\int \frac{1}{a-b\,\text{Cosh}[x]^5}\,dx$$

Optimal (type 3, 494 leaves, 12 steps):

$$\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}+b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}-b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-b^{1/5}}\,\sqrt{a^{1/5}+b^{1/5}}}+\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}-(-1)^{1/5}b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}+(-1)^{1/5}b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-(-1)^{1/5}b^{1/5}}\,\sqrt{a^{1/5}+(-1)^{1/5}b^{1/5}}}+\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}+(-1)^{2/5}b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}-(-1)^{2/5}b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-(-1)^{2/5}b^{1/5}}\,\sqrt{a^{1/5}+(-1)^{2/5}b^{1/5}}}+$$
$$\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}-(-1)^{3/5}b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}+(-1)^{3/5}b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-(-1)^{3/5}b^{1/5}}\,\sqrt{a^{1/5}+(-1)^{3/5}b^{1/5}}}+\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}+(-1)^{4/5}b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}-(-1)^{4/5}b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-(-1)^{4/5}b^{1/5}}\,\sqrt{a^{1/5}+(-1)^{4/5}b^{1/5}}}$$

Result (type 7, 139 leaves):

$$-\frac{8}{5}\,\text{RootSum}\Big[b+5\,b\,\#1^2+10\,b\,\#1^4-32\,a\,\#1^5+10\,b\,\#1^6+5\,b\,\#1^8+b\,\#1^{10}\,\&,\ \frac{x\,\#1^3+2\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^3}{b+4\,b\,\#1^2-16\,a\,\#1^3+6\,b\,\#1^4+4\,b\,\#1^6+b\,\#1^8}\,\&\Big]$$

## Problem 68: Result is not expressed in closed-form.

$$\int \frac{1}{a-b\,\text{Cosh}[x]^6}\,dx$$

Optimal (type 3, 175 leaves, 7 steps):

$$\frac{\text{ArcTanh}\left[\frac{a^{1/6}\,\text{Tanh}[x]}{\sqrt{a^{1/3}-b^{1/3}}}\right]}{3\,a^{5/6}\,\sqrt{a^{1/3}-b^{1/3}}}+\frac{\text{ArcTanh}\left[\frac{a^{1/6}\,\text{Tanh}[x]}{\sqrt{a^{1/3}+(-1)^{1/3}\,b^{1/3}}}\right]}{3\,a^{5/6}\,\sqrt{a^{1/3}+(-1)^{1/3}\,b^{1/3}}}+\frac{\text{ArcTanh}\left[\frac{a^{1/6}\,\text{Tanh}[x]}{\sqrt{a^{1/3}-(-1)^{2/3}\,b^{1/3}}}\right]}{3\,a^{5/6}\,\sqrt{a^{1/3}-(-1)^{2/3}\,b^{1/3}}}$$

Result (type 7, 132 leaves):

$$-\frac{16}{3}\,\text{RootSum}\left[b+6\,b\,\#1+15\,b\,\#1^2-64\,a\,\#1^3+20\,b\,\#1^3+15\,b\,\#1^4+6\,b\,\#1^5+b\,\#1^6\,\&,\ \frac{x\,\#1^2+\text{Log}[-\text{Cosh}[x]-\text{Sinh}[x]+\text{Cosh}[x]\,\#1-\text{Sinh}[x]\,\#1]\,\#1^2}{b+5\,b\,\#1-32\,a\,\#1^2+10\,b\,\#1^2+10\,b\,\#1^3+5\,b\,\#1^4+b\,\#1^5}\,\&\right]$$

## Problem 69: Result is not expressed in closed-form.

$$\int \frac{1}{a-b\,\text{Cosh}[x]^8}\,dx$$

Optimal (type 3, 213 leaves, 9 steps):

$$\frac{\text{ArcTanh}\left[\frac{a^{1/8}\,\text{Tanh}[x]}{\sqrt{a^{1/4}-b^{1/4}}}\right]}{4\,a^{7/8}\,\sqrt{a^{1/4}-b^{1/4}}}+\frac{\text{ArcTanh}\left[\frac{a^{1/8}\,\text{Tanh}[x]}{\sqrt{a^{1/4}-i\,b^{1/4}}}\right]}{4\,a^{7/8}\,\sqrt{a^{1/4}-i\,b^{1/4}}}+\frac{\text{ArcTanh}\left[\frac{a^{1/8}\,\text{Tanh}[x]}{\sqrt{a^{1/4}+i\,b^{1/4}}}\right]}{4\,a^{7/8}\,\sqrt{a^{1/4}+i\,b^{1/4}}}+\frac{\text{ArcTanh}\left[\frac{a^{1/8}\,\text{Tanh}[x]}{\sqrt{a^{1/4}+b^{1/4}}}\right]}{4\,a^{7/8}\,\sqrt{a^{1/4}+b^{1/4}}}$$

Result (type 7, 158 leaves):

$$-16\,\text{RootSum}\Big[b+8\,b\,\#1+28\,b\,\#1^2+56\,b\,\#1^3-256\,a\,\#1^4+70\,b\,\#1^4+56\,b\,\#1^5+28\,b\,\#1^6+8\,b\,\#1^7+b\,\#1^8\,\&,$$
$$\frac{x\,\#1^3+\text{Log}[-\text{Cosh}[x]-\text{Sinh}[x]+\text{Cosh}[x]\,\#1-\text{Sinh}[x]\,\#1]\,\#1^3}{b+7\,b\,\#1+21\,b\,\#1^2-128\,a\,\#1^3+35\,b\,\#1^3+35\,b\,\#1^4+21\,b\,\#1^5+7\,b\,\#1^6+b\,\#1^7}\,\&\Big]$$

## Problem 70: Result is not expressed in closed-form.

$$\int \frac{1}{1+\text{Cosh}[x]^5}\,dx$$

Optimal (type 3, 223 leaves, 11 steps):

$$-\frac{2\,\text{ArcTan}\left[\frac{\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{-\frac{1-(-1)^{1/5}}{1+(-1)^{1/5}}}}\right]}{5\sqrt{-1+\left(-1\right)^{2/5}}}-\frac{2\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\,\text{ArcTan}\left[\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\,\text{Tanh}\left[\frac{x}{2}\right]\right]}{5\left(1+\left(-1\right)^{3/5}\right)}+$$

$$\frac{2\,\text{ArcTanh}\left[\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}\,\text{Tanh}\left[\frac{x}{2}\right]\right]}{5\sqrt{1-\left(-1\right)^{4/5}}}+\frac{2\,\text{ArcTanh}\left[\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}}\,\text{Tanh}\left[\frac{x}{2}\right]\right]}{5\sqrt{1+\left(-1\right)^{3/5}}}+\frac{\text{Sinh}[x]}{5\left(1+\text{Cosh}[x]\right)}$$

Result (type 7, 445 leaves):

$$-\frac{1}{10}\,\text{RootSum}\Big[1-2\,\#1+8\,\#1^2-14\,\#1^3+30\,\#1^4-14\,\#1^5+8\,\#1^6-2\,\#1^7+\#1^8\,\&,$$
$$\frac{1}{-1+8\,\#1-21\,\#1^2+60\,\#1^3-35\,\#1^4+24\,\#1^5-7\,\#1^6+4\,\#1^7}\Big(x+2\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]-4\,x\,\#1-$$
$$8\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1+15\,x\,\#1^2+30\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^2-$$
$$40\,x\,\#1^3-80\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^3+15\,x\,\#1^4+$$
$$30\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^4-4\,x\,\#1^5-8\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^5+$$
$$x\,\#1^6+2\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^6\Big)\,\&\Big]+\frac{1}{5}\,\text{Tanh}\left[\frac{x}{2}\right]$$

## Problem 72: Result is not expressed in closed-form.

$$\int\frac{1}{1+\text{Cosh}[x]^8}\,dx$$

Optimal (type 3, 129 leaves, 9 steps):

$$\frac{\text{ArcTanh}\left[\frac{\text{Tanh}[x]}{\sqrt{1-(-1)^{1/4}}}\right]}{4\sqrt{1-\left(-1\right)^{1/4}}}+\frac{\text{ArcTanh}\left[\frac{\text{Tanh}[x]}{\sqrt{1+(-1)^{1/4}}}\right]}{4\sqrt{1+\left(-1\right)^{1/4}}}+\frac{\text{ArcTanh}\left[\frac{\text{Tanh}[x]}{\sqrt{1-(-1)^{3/4}}}\right]}{4\sqrt{1-\left(-1\right)^{3/4}}}+\frac{\text{ArcTanh}\left[\frac{\text{Tanh}[x]}{\sqrt{1+(-1)^{3/4}}}\right]}{4\sqrt{1+\left(-1\right)^{3/4}}}$$

Result (type 7, 127 leaves):

$$16\,\text{RootSum}\left[1+8\,\#1+28\,\#1^2+56\,\#1^3+326\,\#1^4+56\,\#1^5+28\,\#1^6+8\,\#1^7+\#1^8\,\&,\ \frac{x\,\#1^3+\text{Log}\left[-\text{Cosh}[x]-\text{Sinh}[x]+\text{Cosh}[x]\,\#1-\text{Sinh}[x]\,\#1\right]\#1^3}{1+7\,\#1+21\,\#1^2+163\,\#1^3+35\,\#1^4+21\,\#1^5+7\,\#1^6+\#1^7}\,\&\right]$$

## Problem 73: Result is not expressed in closed-form.

$$\int \frac{1}{1-\text{Cosh}[x]^5}\,dx$$

Optimal (type 3, 205 leaves, 11 steps):

$$-\frac{2\,\text{ArcTan}\left[\frac{\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{-\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}}\right]}{5\sqrt{-1+(-1)^{4/5}}}+\frac{2\,\text{ArcTan}\left[\sqrt{-\frac{1+(-1)^{4/5}}{1-(-1)^{4/5}}}\,\text{Tanh}\left[\frac{x}{2}\right]\right]}{5\sqrt{-1-(-1)^{3/5}}}+\frac{2\,\text{ArcTanh}\left[\sqrt{\frac{1-(-1)^{1/5}}{1+(-1)^{1/5}}}\,\text{Tanh}\left[\frac{x}{2}\right]\right]}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\,\text{ArcTanh}\left[\sqrt{\frac{1-(-1)^{3/5}}{1+(-1)^{3/5}}}\,\text{Tanh}\left[\frac{x}{2}\right]\right]}{5\sqrt{1+(-1)^{1/5}}}-\frac{\text{Sinh}[x]}{5\,(1-\text{Cosh}[x])}$$

Result (type 7, 445 leaves):

$$\frac{1}{5}\text{Coth}\left[\frac{x}{2}\right]+\frac{1}{10}\text{RootSum}\Big[1+2\,\#1+8\,\#1^2+14\,\#1^3+30\,\#1^4+14\,\#1^5+8\,\#1^6+2\,\#1^7+\#1^8\,\&,\ \frac{1}{1+8\,\#1+21\,\#1^2+60\,\#1^3+35\,\#1^4+24\,\#1^5+7\,\#1^6+4\,\#1^7}$$
$$\Big(x+2\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]+4\,x\,\#1+8\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1+15\,x\,\#1^2+$$
$$30\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^2+40\,x\,\#1^3+80\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^3+$$
$$15\,x\,\#1^4+30\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^4+4\,x\,\#1^5+$$
$$8\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^5+x\,\#1^6+2\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^6\Big)\,\&\Big]$$

## Problem 81: Result is not expressed in closed-form.

$$\int \frac{\text{Tanh}[x]^3}{a+b\,\text{Cosh}[x]^3}\,dx$$

Optimal (type 3, 153 leaves, 11 steps):

$$-\frac{b^{2/3}\,\text{ArcTan}\left[\frac{a^{1/3}-2\,b^{1/3}\,\text{Cosh}[x]}{\sqrt{3}\,a^{1/3}}\right]}{\sqrt{3}\,a^{5/3}}+\frac{\text{Log}[\text{Cosh}[x]]}{a}+\frac{b^{2/3}\,\text{Log}\left[a^{1/3}+b^{1/3}\,\text{Cosh}[x]\right]}{3\,a^{5/3}}-$$
$$\frac{b^{2/3}\,\text{Log}\left[a^{2/3}-a^{1/3}\,b^{1/3}\,\text{Cosh}[x]+b^{2/3}\,\text{Cosh}[x]^2\right]}{6\,a^{5/3}}-\frac{\text{Log}\left[a+b\,\text{Cosh}[x]^3\right]}{3\,a}+\frac{\text{Sech}[x]^2}{2\,a}$$

Result (type 7, 145 leaves):

$$\frac{1}{6a}\left(-6x+6\,\text{Log}[\text{Cosh}[x]]-2\,\text{RootSum}\left[b+3b\,\#1^2+8a\,\#1^3+3b\,\#1^4+b\,\#1^6\,\&,\ \frac{-b\,x+b\,\text{Log}[e^x-\#1]-4a\,x\,\#1^3+4a\,\text{Log}[e^x-\#1]\,\#1^3-3b\,x\,\#1^4+3b\,\text{Log}[e^x-\#1]\,\#1^4}{b+2b\,\#1^2+4a\,\#1^3+b\,\#1^4}\,\&\right]+3\,\text{Sech}[x]^2\right)$$

## Problem 82: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]}{\sqrt{a+b\,\text{Cosh}[x]^3}}\,dx$$

Optimal (type 3, 28 leaves, 4 steps):

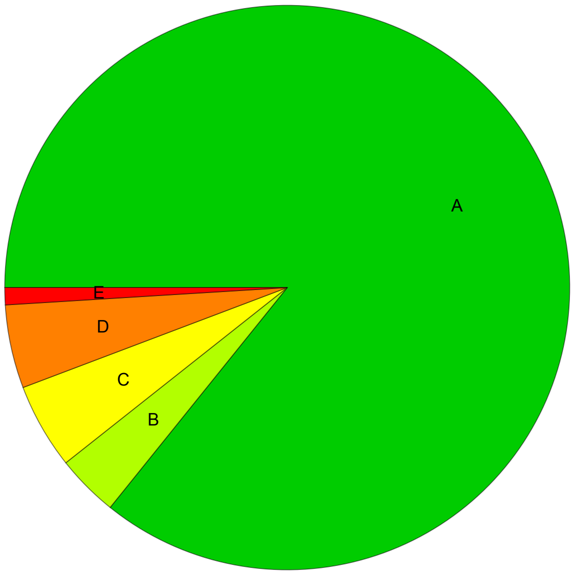
$$-\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Cosh}[x]^3}}{\sqrt{a}}\right]}{3\sqrt{a}}$$

Result (type 3, 66 leaves):

$$-\frac{2\sqrt{b}\,\text{ArcSinh}\left[\frac{\sqrt{a}\,\text{Sech}[x]^{3/2}}{\sqrt{b}}\right]\sqrt{\frac{b+a\,\text{Sech}[x]^3}{b}}}{3\sqrt{a}\,\sqrt{a+b\,\text{Cosh}[x]^3}\,\text{Sech}[x]^{3/2}}$$

# Summary of Integration Test Results

816 integration problems




A - 700 optimal antiderivatives

B - 29 more than twice size of optimal antiderivatives

C - 40 unnecessarily complex antiderivatives

D - 39 unable to integrate problems

E - 8 integration timeouts